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THE UNIVERSITY OF ALBERTA

SPECTRAL ANALYSIS OF RELATIVE HUMIDITY AND  
VERTICAL FLUXES OF HEAT AND MOISTURE

by



JAMES E. McGREGOR

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES  
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The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies for acceptance, a thesis entitled "Spectral Analysis of Relative Humidity and Vertical Fluxes of Heat and Moisture", submitted by James E. McGregor in partial fulfilment of the requirements for the degree of Master of Science.



## ABSTRACT

It is a well-known fact that the changes in the mean relative humidity have a profound effect on the development of certain microbes, but it has not been completely established that fluctuations are important in the relationship between relative humidity and microorganisms. To determine the possible relationships, the first problem was to investigate the relative humidity variations that occurred in turbulent air flow near the ground. The variations or fluctuations can be specified by spectral analysis and the first part of this thesis is to relate the spectra to atmospheric parameters. Furthermore, the data obtained by the project should be ideally suited to the calculation of vertical heat and moisture fluxes. Fluxes and eddy transfer coefficients are the second concern of the thesis.

The spectral analysis of relative humidity is divided into two parts; an investigation of the variance and skewness and an examination of the spectral density. It was discovered that the standard deviation was related to the vertical gradient, wind speed, wind shear, and a time factor. Linear combinations of standard deviations and skewnesses failed to reveal any plausible coefficients. The sign of the relative humidity skewness was found to depend on the stability. The spectral density, shown as a graph of normalized variance and non-dimensional frequency, exhibits a bimodal curve for unstable and neutral trials. The low frequency or convective peak is understandably absent from the stable trials. The high frequency maximum, caused by mechanical turbulence, is present in all stability cases. The relative importance of the peaks depends on such factors as wind speed and vertical gradients. A comparison of temperature and specific humidity spectra with relative humidity spectra



shows some striking similarities under certain conditions. This suggests that temperature or specific humidity dominates in controlling the relative humidity. The individual sharp peaks of the spectra do not exceed the calculated confidence limits and, therefore, they are not considered to be significant. Consecutive trials and increased sample length reveal reproducibility and stability of a running mean of the spectra and a lack of these characteristics for the individual peaks. The problem of aliasing is serious in the calculated spectra and its effect must be kept in mind at all times. The failure of relative humidity spectra to obey the minus five-thirds power law is attributed to aliasing. Using the standard deviation relationships and the behaviour of the relative humidity spectra, a general specification of the relative humidity fluctuations can be made from steady state parameters.

The calculated fluxes and coefficients of conductivity and diffusivity are small in number and low in accuracy. The poor quality of the specific humidity gradient introduces an unknown error into the coefficient of diffusivity. These deficiencies are too serious to permit one to draw any reliable conclusions.

The spectral analysis and the flux calculations are useful in making inferences concerning relative humidity fluctuations and the eddy transfer coefficients. For future research, suggestions are made to improve the instruments and the methods.



## ACKNOWLEDGEMENTS

During the summer of 1967, I was stationed at Defense Research Establishment Suffield as an employee of the Meteorological Branch of the Department of Transport. The data used in this thesis were obtained with instruments and personnel supplied by DRES. Analysis of the data was performed at the University of Alberta from September, 1967 to July, 1968.

I would like to thank the people at Suffield who made my experiences rewarding and enjoyable. I am grateful to Dr. R.B. Harvey for the initial contact made with me and for his helpful support; to Dr. E.R. Walker who initiated the project; to T. McIntosh who wrote the summary on spectral analysis; and to J.A. McCallum for his constant guidance and helpful discussion. Also, an expression of thanks to B.R. Campbell, H.S. Johnson, O. Johnson, Mrs. B.R. Larson, and H. Lutz for their varied and necessary assistance. They were responsible for the construction and operation of the thermocouple, the recording and digitization of the data, the writing of the data conversion computer programs, and the aid and guidance I received. Gratitude is expressed for the unmentioned individuals who helped in many ways.

The complicated computer programs for the analysis were written by myself. However, without the use of the 360-67 IBM computer at the University of Alberta the analysis could not have been done. I am grateful for the use of the computer and the other services provided by the Department of Computing Science.

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## TABLE OF CONTENTS

	Page
ABSTRACT	iii
ACKNOWLEDGEMENTS	v
TABLE OF CONTENTS	vii
LIST OF TABLES	ix
LIST OF ILLUSTRATION	x
 Chapter	
I. OUTLINE OF THE PROJECT	1
Problems	1
Historical	2
Instrumentation and Data	3
Reasons and Aims	4
II. INSTRUMENTATION AND THE RAW DATA	7
Thermocouples	8
Vectorvane	14
Auxiliary Instruments	15
Accumulated Errors	16
Site and Instrument Layout	17
Raw Data	17
III. MODIFICATION AND CONVERSION OF THE RAW DATA	18
The Data Period	18
Data Modification	19
The Time Series	21
IV. BASIC ANALYSIS AND OBJECTIVES	25
Spectral Analysis	25
Flux Calculations	31
V. RELATIVE HUMIDITY THEORY	34
Standard Deviation	34
Skewness	40
Spectral Characteristics	41
Spectral Relations	44
Summary	45



VI. RELATIVE HUMIDITY ANALYSIS	46
Standard Deviation	46
Skewness	51
Spectral Characteristics	53
Spectral Relations	60
Conclusions	62
VII. FLUX ANALYSIS	63
VIII. CONCLUSIONS	68
Relative Humidity Spectral Analysis	68
Heat and Moisture Fluxes	69
Suggestions for Future Research	69
Summary	71
BIBLIOGRAPHY	72
APPENDIX A	77
APPENDIX B	86
APPENDIX C	87
APPENDIX D	101
APPENDIX E	105



LIST OF TABLES

Table	Page
I. Values of a for Equation (5.1)	47
II. Constant b of Equation (5.8)	48
III. Constant c of the Similarity Equation (5.13)	49
IV. Linear Coefficients of Standard Deviations	50
V. Ratio of Standard Deviation and Maximum One Second Change	51
VI. Skewness: Sign Relation and Linear Coefficients	52
VII. Skewness Sign and Extreme Departures from the Mean	53
VIII. Power (Slope) of the Spectral Power Law	59
IX. Frequency Range for Relative Humidity and Both Temperature and Specific Humidity.	61
X. Eddy Fluxes and Coefficients	64
XI. Data for Calculation of Coefficients	64
XII. Means of the Data	87
XIII. Multiplication Constants for Upper and Lower Confidence Limits	105



## LIST OF ILLUSTRATIONS

Illustration	after Page
1. Thermocouples	9
2. Vectorvane	14
3. Instrument Layout	17



## CHAPTER I

### OUTLINE OF THE PROJECT

Turbulence studies of the last two decades have been wide in scope and varied in approach. In both practical and theoretical work, advancement has been steady. However, measurement of some meteorological parameters which are important to other scientific fields has lagged. One such property is the relative humidity, which is a prime interest of microbiology. A second quantity is the simultaneous measurement of the vertical fluxes of heat and moisture. Considerable work has been done with heat flux but the lack of a suitable sensor has slowed the measurement of moisture flux. These two topics are to be discussed in this thesis.

#### Problems

Fluctuations of relative humidity at a fixed point in space are a consequence of the turbulent flow of air. The changes in the temperature and in the specific humidity cause related changes in the relative humidity. These variations can be described by their amplitude and their frequency. The amplitude is determined by the variance, skewness, and other moments. The division of the variance among certain frequency bands is accomplished by spectral analysis. Therefore, a specification of the relative humidity fluctuations can be obtained from statistical properties and spectral analysis.

Vertical fluxes can be measured by fast-response instruments. If the temperature, specific humidity, and vertical wind are recorded simultaneously, the calculated fluxes can be used to determine eddy transfer coefficients of conductivity and diffusivity. Accurate measurement of the vertical gradients of temperature and specific



humidity are required to give reliable estimates for the coefficients. The coefficients and their ratio are important to the problem of the transfer mechanism of different quantities.

### Historical

Spectral analysis is a well established procedure. Its application to meteorological data is based mainly on techniques developed by Blackman and Tukey<sup>1</sup>. Clarification and improvement of the theory have been done by Jenkins<sup>2</sup> and Panofsky and Brier<sup>3</sup>. A summary of experimental results and spectra is given by Lumley and Panofsky<sup>4</sup>. Spectra of turbulent parameters have been obtained by Webb<sup>5</sup>, Smith<sup>6</sup>,

<sup>1</sup>R.B. Blackman and J.W. Tukey, The Measurement of Power Spectra, Dover Publications, New York, 1958, pp. 1-190.

<sup>2</sup>G.M. Jenkins, "General Considerations in the Analysis of Spectra"., Technometrics, Vol. 3, No. 2, 1961, pp. 133-166. and "A Survey of Spectral Analysis", Applied Statistics, Vol. 14, No. 1, 1965, pp. 2-32.

<sup>3</sup>H.A. Panofsky and G.W. Brier, Some Application of Statistics to Meteorology, Pennsylvania State University, University Park, Penn. 1965, pp. 1-224.

<sup>4</sup>J.L. Lumley and H.A. Panofsky, The Structure of Atmospheric Turbulence, John Wiley & Sons, New York, 1964, pp. 1-239.

<sup>5</sup>E.K. Webb, Autocorrelation and Energy Spectra of Atmospheric Turbulence, C.S.I.R.O., Div. Met. Phys. Tech. Pap. No. 5, Melbourne, Australia, 1955, pp. 1-28.

<sup>6</sup>S.D. Smith, "Thrust-anemometer Measurements of Wind-velocity Spectra and of Reynolds Stress over a Coastal Inlet: Journ. of Marine Research, Vol. 25, No. 3, 1967, pp. 239-262, and Thrust Anemometer Measurement of Wind Velocity Fluctuations, Spectra, and Stress Over the Sea, unpublished manuscript, Bedford Institute of Oceanography, Dartmouth, N.S., report BIO 66-8, 1966, pp. 1-153.



Loucks<sup>7</sup>, and Elagina<sup>8</sup>. The conclusions of these projects are important in planning a set of experiments and in the comparison of final results.

The measurement of vertical fluxes by fast-response instruments was initiated by Swinbank<sup>9</sup> and followed up by McIlroy<sup>10</sup> and Dyer and Pruitt<sup>11</sup>. Similar work has been done by Högström<sup>12</sup> and Hay<sup>13</sup>. The calculated heat and moisture fluxes of these experiments are useful as independent checks on the range of possible flux values.

#### Instrumentation and Data

The Defense Research Establishment Suffield provided the instrumentation and personnel required to collect the data used for the

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<sup>7</sup>R.H. Loucks, A Record of Some Measurements of Atmospheric Turbulence over Water, unpublished manuscript, Bedford Institute of Oceanography, Dartmouth, N.S., Internal note 1966-10-I, 1966, pp. 1-59.

<sup>8</sup>L.G. Elagina, "Measurement of Frequency Spectra of Pulsations of Absolute Humidity in the Ground Layer of the Atmosphere", Inst. Fiz. Atm., ANSSSR, Moscow, 1964, pp. 237-244.

<sup>9</sup>W.C. Swinbank, "The Measurement of Vertical Transfer of Heat and Water Vapour by Eddies in the Lower Atmosphere", Journ. of Met., Vol. 8, 1951, pp. 135-145 and An Experimental Study of Eddy Transports in the Lower Atmosphere, C.S.I.R.O., Div. Met. Phys. Pap. No. 2, Melbourne, Australia, 1955, pp. 1-30.

<sup>10</sup>I.C. McIlroy, Effects of Instrumental Response on Atmospheric Flux Measurement by the Eddy-correlation Method C.S.I.R.O., Div. Met. Phys. Pap. No. 11, Melbourne, Australia, 1961, pp. 1-30.

<sup>11</sup>A.J. Dyer and W.O. Pruitt, "Eddy Flux Measurements Over a Small Irrigated Area", Journ. of Appl. Met., Vol. 1, 1962, p. 471-473.

<sup>12</sup>U. Högström, "A New Sensitive Eddy Flux Instrumentation", Tellus, Vol. 19, No. 2, 1967, pp. 230-239.

<sup>13</sup>D.R. Hay et al., "On the Eddy Transfer of Water Vapour Above an Outdoor Surface", in A. Wexler (ed.), Humidity and Moisture, Reinhold Publishers, New York, 1965, Vol. 2, pp. 583-587.



spectral analysis and flux calculations. Nine successful trials were completed during the summer of 1967. The raw data, recorded on magnetic tape, were digitized on paper punch tape which was fed to Suffield's 1130 IBM computer. The final data of temperature, wet bulb temperature, relative humidity, and vertical wind were punched on cards. At the University of Alberta the data were used in the 360-67 IBM computer to calculate the spectral estimates and the vertical fluxes. Some of the output was plotted by the university's Calcomp plotter. A portion of the graphs is shown in Appendix C and Appendix E.

The fast-response instruments used to measure the temperature, humidity, and vertical wind were wet and dry bulb thermocouples and a vectorvane. The sensors were calibrated so that the digitized output could be converted to the most convenient forms.

Mean properties of the atmosphere were measured. These consisted of the temperature, the wet bulb temperature, vertical gradients of temperature and wet bulb, and mean winds at three levels.

The objective of the project was to obtain three sets of data for each of the stability cases; unstable, neutral, and stable. Each set of data consisted of continuously recorded time series of temperature, wet bulb depression, wind speed, and elevation angle of the wind. The mean properties were manually read and recorded at specific intervals throughout each period. The nine trials were digitized, converted, and used for the spectral analysis and flux calculations

#### Reasons and Aims

The effect of relative humidity on microorganisms is quite



marked under certain conditions. This is expressed by Wells<sup>14</sup>, Rosebury<sup>15</sup>, and Flitters<sup>16</sup>. If changes in the mean relative humidity are important, the fluctuations are likely to be important also. Thus, any specifications of the fluctuations would be helpful in analysing results of microbiological experiments. Therefore, the aim of the relative humidity spectral analysis is to determine relationships, if any, between relative humidity fluctuations and the mean properties such as vertical gradients and wind speeds.

The measurement or calculation of vertical transfer by eddies is important in the study of the heat balance of the atmosphere. Simultaneous determination of the heat and moisture fluxes leads to the coefficients of conductivity and diffusivity. A comparison of the coefficients is useful in appraising the transfer mechanism of turbulent air flow near the earth's surface. Swinbank<sup>17</sup> states that,

"... the effect of buoyancy in distinguishing the mechanism of heat transfer from that of other properties has been established beyond reasonable doubt".

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<sup>14</sup>W.F. Wells, Airborne Contagion and Air Hygiene, Harvard University Press, Cambridge, Mass., 1955., pp. 1-423.

<sup>15</sup>T. Rosebury, Experimental Air-Borne Infection, The Williams & Wilkins Co., Baltimore, Md., 1947, pp. 1-222.

<sup>16</sup>N.E. Flitters, "Programming Relative Humidity Combination with Fluctuating Temperature: The Influence of Relative Humidity on Developement of Tropical Fruit Flies and Other Insects", in A. Wexler (ed.), Humidity and Moisture, Reinhold Publishers, New York, 1965, Vol. 2, pp. 65-72.

<sup>17</sup>W.C. Swinbank, An Experimental Study of Eddy Transports in the Lower Atmosphere, C.S.I.R.O., Div. Met. Phys. Pap. No. 2, Melbourne, Australia, 1955, p. 24.



However, additional measurements and analysis have resulted in a change of view. Swinbank and Dyer<sup>18</sup> and Dyer<sup>19</sup> show that the coefficients of conductivity and diffusivity are very similar and with certain corrections can be equal. Crawford<sup>20</sup> agrees with the conclusion. Also, Munn<sup>21</sup> shows that the ratio of the coefficients is near unity. Considering the recent work, it is reasonable to assume equality of the coefficients of conductivity and diffusivity. Therefore, the aim of the flux calculations is to determine the eddy transfer coefficients, examine their ratio, and investigate its dependence on other factors.

It is hoped that the investigation of relative humidity fluctuations and heat and moisture fluxes will be useful for the initiation of new ideas and clarification of old hypotheses.

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<sup>18</sup>W.C. Swinbank and A.J. Dyer, "An Experimental Study in Micrometeorology", Quart. Journ. of Roy. Met. Soc., Vol. 93, 1967, pp. 499-500.

<sup>19</sup>A.J. Dyer, "The Turbulent Transport of Heat and Water Vapour in an Unstable Atmosphere", Quart. Journ. of Roy. Met. Soc. Vol. 93, 1967, p. 507.

<sup>20</sup>T.V. Crawford, "Moisture Transfer in Free and Forced Convection", Quart. Journ. of Roy. Met. Soc., Vol. 91, 1965, p. 26.

<sup>21</sup>R.E. Munn, Descriptive Micrometeorology, Academic Press, New York, Advances in Geophysics, Supplement 1, 1966, pp. 94-95.



## CHAPTER II

### INSTRUMENTATION AND THE RAW DATA

Measurement of atmospheric parameters in the field at undetermined times over a period of several months requires instruments which are sturdy, replaceable, unaffected by time drift, and easily maintained. Also, quick installation at the site and the capacity for a wide range of values are desirable. In order to measure high frequency fluctuations, fast-response instruments are a necessity. Furthermore, an accurate determination of humidity depends on the time constants<sup>1</sup> of the wet and dry bulb thermometers. Using sinusoidal fluctuations, Taylor<sup>2</sup> showed that the product of the maximum frequency in the analysis and the wet bulb time constant must be less than one. When this product is less than one half, as it is in the cases to be studied, the ratio of the time constants should be near unity to achieve a good degree of accuracy in the calculation of humidity.

These factors were considered when small thermocouples were chosen to measure the temperature and the wet bulb temperature. The thermocouples were constructed and preliminary tests showed that they fulfilled most of the requirements. The time constants of the wet and dry junctions were found to be closely matched.

The vertical wind component, to be used in the flux and spectral

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<sup>1</sup>Time constant is defined as the time required for an instrument to respond to 63.2% ( $1-1/e$ ) of a discrete change in the environment for a given flow rate.

<sup>2</sup>R.J. Taylor, "The Response of a Psychrometer to Fluctuations in Vapour Pressure", in A. Wexler (ed.), Humidity and Moisture, Reinhold Publishers, New York, Vol. 1, 1965, pp. 80-81.



calculations, can be determined from the wind speed and its elevation angle. To measure these quantities a vectorvane was purchased from Meteorological Research Incorporated (MRI). The vectorvane, when properly maintained and calibrated, was well suited to the conditions in which it was used. In the range of wind speeds from three to eight metres per sec. the vectorvane time constant was closely matched with those of the wet and dry bulb thermocouple.

Vertical wind shear, vertical gradients of humidity and temperature, and two-metre mean values of the temperature, the wet bulb, and the wind were measured at regular intervals throughout each trial. The mean values of wet bulb depression, temperature, and wind were combined with the fluctuations to yield the raw data time series.

#### Thermocouples

The two-metre thermocouples were constructed of copper and constantan wire, .003 inches in diameter. The junctions, formed by welding the ends together, had slightly greater diameters. The dry thermocouple was coated with plastic to increase its time constant so that it matched that of the wet junction. Fine cotton thread was wound circularly about the axis of the wet junction and its leads for a distance of two to three inches. This increased the diameter of the wet junction to about .006 inches and when wet the effective diameter was estimated to be .008 inches. The leads to both junctions were placed in glass tubes to hold the junctions away from the assembly. Furthermore, the tubes acted as a reservoir for each end of the wick on the wet junction. The thermocouples were not shielded from solar radiation. A third thermocouple, packed in insulation and covered with metal foil, was used as a reference.



Its time constant of twenty minutes was sufficiently long to assume a constant or linearly varying reference for any ten minute period.

The dry junction was referenced to this insulated junction and the wet junction was referenced to the dry. Thus, the output voltages of the thermocouples corresponded to a relative temperature and to the wet bulb depression. A picture of the thermocouples is shown in Illustration 1.

Because the output voltages are very small, twenty-two microvolts per F degree difference between two junctions, they must be amplified to be recorded. The magnetic tape recorder range was set at plus or minus 1.4 volts. The temperature and wet bulb depression fluctuations were assumed to fall within a plus or minus five degree F range. This required an amplification from 110 microvolts (corresponding to five degrees F) to 1.4 volts, which is approximately 13000 times. This was achieved by a pair of amplifiers for both the temperature and depression voltages. However, the depression was expected to exceed five degrees F so a feed back circuit was developed to reduce the amplified signal. On occasions the relative temperature also exceeded the five degree F limit. Consequently, a feed back was applied to the temperature voltage when it was required. Therefore, the temperature and depression fluctuations, recorded on two of the fourteen channels of magnetic tape, are just fluctuations about an arbitrary constant.

Tests conducted on the thermocouples determined their main characteristics. Most important of these are the time constants. For decreasing temperatures they were found to be about 0.35 seconds for the dry and 0.40 seconds for the wet junction in a wind of 2.3 metres per sec. The time constants were about 0.1 seconds faster for increasing



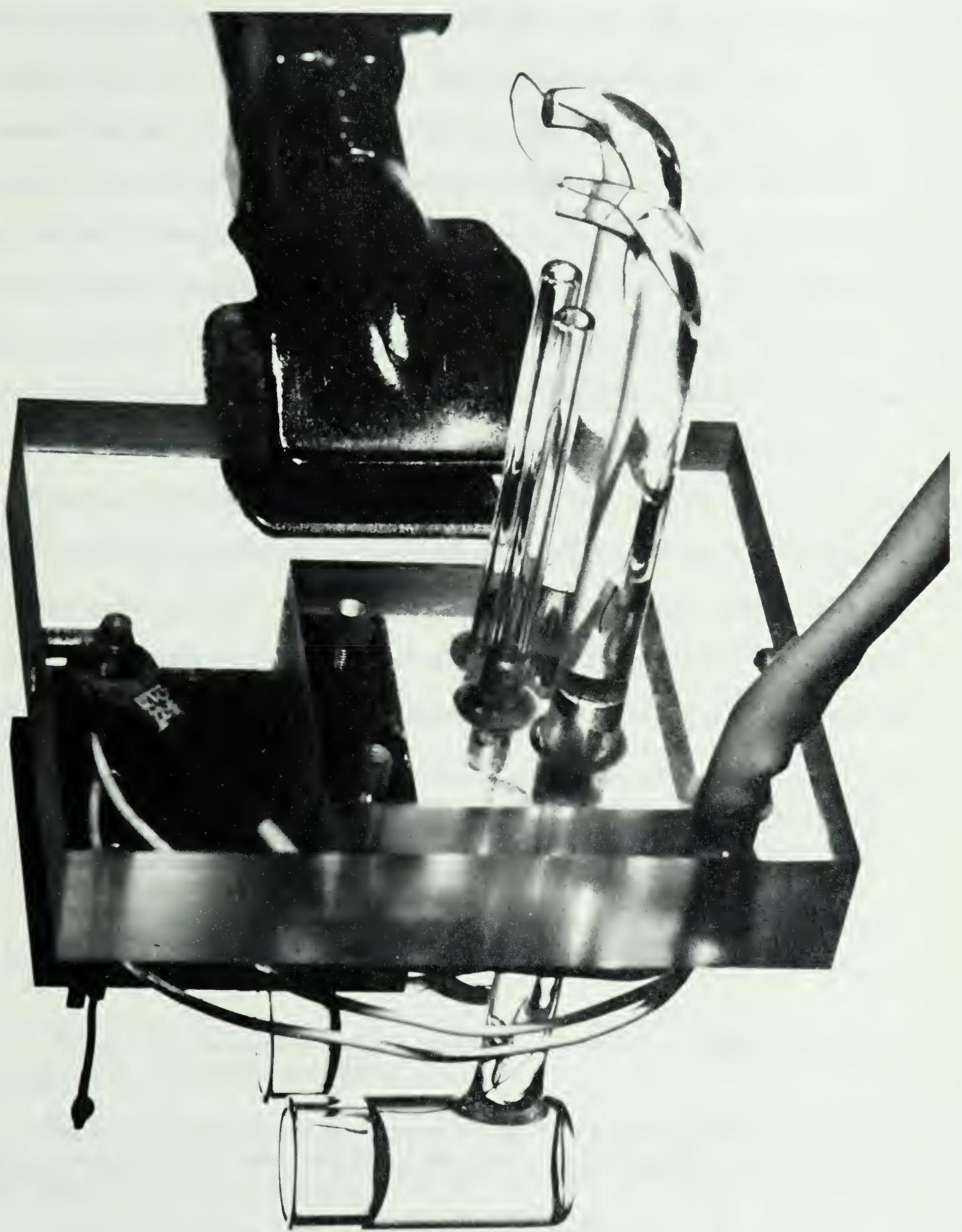


Illustration 1 - Thermocouples



temperatures. When the wind was increased to 4.6 metres per sec. the time constants decreased to approximately 0.30 and 0.35 seconds for decreases and 0.15 and 0.20 seconds for increasing temperatures<sup>3</sup>.

Swinbank<sup>4</sup>, using a somewhat finer thermocouple for the dry bulb, gives a table of period of fluctuation versus phase lags. From these, the time constants were calculated to be 0.4 seconds for a wind of one metre per sec. and 0.2 seconds for a four metre per sec. wind. The .003 inch diameter wet junction used by Swinbank<sup>5</sup> had a time constant of 0.3 seconds and was independent of wind speed. Sheppard and Elnesr<sup>6</sup> determined the time constants for wet and dry thermocouples constructed of .005 inch diameter wire to be 0.2 and 0.13 seconds in a 5 metre per sec. wind. Taylor<sup>7</sup> derived an equation to calculate the time constant of a wet wire. For a two metre per sec. wind, a diameter of .008 inches, and a mean wet bulb temperature of ten degrees C, the time constant of a wet thermocouple would be 0.3 seconds. Considering the range of the time constants

<sup>3</sup>B.R. Campbell, Fast Response Thermocouple System, unpublished paper, Defense Research Establishment Suffield, Ralston, Alberta, 1967, pp. 1-13.

<sup>4</sup>W.C. Swinbank, "The Measurement of Vertical Transfer of Heat and Water Vapour by Eddies in the Lower Atmosphere", Journ. of Met., Vol. 8, No. 3, 1951, p. 139.

<sup>5</sup>Idem, "A Sensitive Vapour Pressure Recorder", Journ. of Sci. Instrum., Vol. 28, 1951, pp. 86-89.

<sup>6</sup>P.A. Sheppard and M.K. Elnesr, "On the Direct Measurement of Humidity Mixing Ratio and its Fluctuations", Quart. Journ. of Roy. Met. Soc., Vol. 77, 1951, p. 452.

<sup>7</sup>Taylor, op. cit., p. 79.



of other thermocouples and the plastic coating, the values determined for the time constants of the thermocouples used in this study are quite realistic. With these values and a maximum frequency to be studied of one half cycle per sec. the condition of matched time constants for accurate humidity calculation is met.

Errors in the measurement of the actual temperature and wet bulb depression come from two sources: the thermocouples (fluctuations) and the psychrometer (means). In flux and spectral calculations the fluctuations are of prime importance. Therefore, the thermocouple errors can be significant. However, errors in the mean values can contribute fluctuation errors when used to calculate the density or humidity series. The errors in the mean values will be discussed later in this chapter.

A systematic error in the thermocouples caused the temperature and wet bulb depression fluctuations to be reduced. This error was caused by the thermocouple resistances being large fractions of the grounding resistances of the amplifiers. The reductions were independent of frequency. Thus, the correction factors, 1.20 and 1.35, were applied to all fluctuations of temperature and depression respectively.

Thermocouple errors due to amplification and recording were minimized by calibrating the signals before and after each trial. Small errors in the thermocouple output, when amplified 13000 times, can become significant. Other large errors were caused by interference on the amplifiers when radio transmissions were made from the site and by a switching device used to monitor different recorder input signals. The latter errors were removed by eliminating the causes. The remaining detectable errors were removed in the data modification process



which is discussed in the next chapter. The smaller, short-lived errors are very nearly averaged out when the magnetic tapes are digitized. Since the thermocouples were unshielded an error due to the absorption of solar radiation can occur. Swinbank<sup>8</sup> estimated the fluctuation error to be about 0.1 degree C when the wind changed from 0.5 metres per sec. to 2.0 metres per sec. In stronger winds this error is much less. One error inherent in the system that can not be corrected for is that caused by the response of the thermocouples to high frequency fluctuations. The measured amplitude is always less than the actual amplitude. The different response times for increasing and decreasing temperatures cause this error to be greatest for decreasing temperatures. For one half cycle per sec. and the stated time constants this error is assumed small and at lower frequencies the error is nonexistent. Neglecting the large random errors which can be removed, the combined error in the thermocouple fluctuations is estimated to be less than 0.02 degrees C.

Characteristics of the wet thermocouple can cause further error in the wet bulb temperature. Poor contact between the water and the junction, insufficient wetting of the wick, and heat conduction from the leads are important factors. The first of these can cause spurious voltages but they are hard to detect and eliminate. The second and third factors compete against one another. To have sufficient wetting of the junction a minimum of wick exposure is desired. This allows heat conduction along the leads to the junction so the wick exposure must be increased. Consequently, 0.75 inches of wick were exposed to the air

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<sup>8</sup>Swinbank, Journ. of Met., op. cit., p. 140



flow. Powell<sup>9</sup>, using similar thermocouples, found that two cm. of exposure was sufficient to attain the maximum wet bulb depression in still air. Also, Swinbank<sup>10</sup> found the depression to differ only 0.2 degrees F from that obtained by an Assman psychrometer. Tests conducted on the wet junction showed that the full depression was not reached. However, the 1.35 correction factor discussed previously must be applied to the depression. The final difference between the psychrometer and thermocouple depression was less than one degree F. This error in total depression should have only a small effect on the fluctuation. With these additional errors the wet bulb fluctuations are estimated to be accurate to 0.03 degrees C.

These temperature errors cause errors when the density and the humidity are calculated. The density error is extremely small. The relative humidity can be in error by 0.2 per cent and the specific humidity by 0.04 g. per kg. Also to be considered is the error in the vapour pressure from the wet and dry junction time constants. For the thermocouples used, this error is less than two per cent of the amplitude of the vapour pressure fluctuation. Combining the amplitude and temperature errors, the relative humidity variations should be accurate to 0.3 per cent and the specific humidity fluctuations to 0.05 g. per kg. For low frequencies these errors would reduce to 0.2 per cent and 0.04 gm. per kg. It should be noted that these errors do not include those incurred

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<sup>9</sup>R.W. Powell, "The Use of Thermocouples for Psychrometric Purposes", Proc. Phys. Soc., Vol. 48, 1936, p. 406.

<sup>10</sup>Swinbank, Journ. of Sci. Instrum., op. cit., p. 86.



from the errors in the mean values.

In view of the difficulty of measuring humidity and the limitations of the thermocouples, fairly accurate results were obtained for use in the calculation of flux and spectra.

#### Vectorvane

The two metre vectorvane used for the trials was the 1053 Model 'MRI vectorvane' shown in Illustration 2. The output consists of three voltages corresponding to azimuth angle, elevation angle, and wind speed. Only the last two are needed to calculate the vertical wind. The two signals were recorded, simultaneously with the temperature and depression, on another two channels of the magnetic tape.

Specifications supplied with the vectorvane gave a distance constant of two to three feet, plus or minus two degrees from linearity for the elevation angle, and plus or minus one per cent from linearity for the speed. The time constant for the vectorvane must be less than one second if the interval in the time series is to be one second. For this reason the absolute minimum wind during a trial was set at one metre per sec. For winds of three and eight metres per sec. the time constants are approximately 0.3 and 0.1 second. Therefore, an attempt was made to have trials when the wind was in this range. For these conditions the wind speed and the sine of the elevation angle are estimated to be accurate to two significant digits. For most of the vertical winds the error involved is less than five cm. per sec. The error in the fluctuations is of the same order. Further errors of unknown origin caused the average wind of the vectorvane to exceed that of the mean wind measured by a cup anemometer. To eliminate most of this error the wind fluctuations





Illustration 2 - Vectorvane.



from the vectorvane were added to the mean wind.

The output of the vectorvane was set so that 1.4 volts was equivalent to 50 m.p.h. and 56 degrees. Conversion factors could be determined from these settings but wind tunnel test and elevation calibration yielded more precise results. The conversion factors were used to change the raw data into suitable units to be used in the flux calculations and spectral analysis.

#### Auxiliary Instruments

A motorized Assman psychrometer was used at the two metre level to obtain mean values of the temperature and wet bulb depression. Readings were taken at five minute intervals and were used as a base line for the temperature and depression fluctuations. The psychrometer was read to the nearest tenth of a degree F but the accuracy was most likely 0.2 degrees F. These errors can cause differences in the humidity and the density but the errors in their fluctuations are small. For 0.2-degree-F errors in the mean wet and dry bulb temperatures, the fluctuations are accurate to 0.0003 gm. per cubic metre for the density, 0.05 per cent for the relative humidity, and 0.0001 gm. per kg. for the specific humidity. In comparison to other errors these can be neglected.

To measure the temperature gradient, two slow-response thermocouples were used at four metres and one half metre. A sensitive galvanometer measured the voltage output which was converted into degrees.

For some of the trials wet bulb mercury thermometers were placed at four metres and one half metre. Like the temperature gradient, these wet bulbs were read at five minute intervals. Using the two metre mean temperature and the temperature and wet bulb gradients, an estimate of



the gradient of humidity was possible. However, the accuracy of this method is poor and the results should only be used as a first estimate. When this method was not available an estimate of the humidity gradient was made using the temperature gradient and available soil moisture as guidelines.

Wind shear was measured by small cup anemometers placed at the two, eight, and sixteen metre levels. The mean two metre wind was also obtained. The winds were averaged over ten or fifteen minute intervals.

Atmospheric pressure was measured by an aneroid barometer to the nearest tenth of a millibar. For an error of 0.2 millibars the error in the density is 0.0002 gm. per cubic metre.

The measurements obtained from the auxiliary instruments are used in the conversion of the raw data, in the calculation of the fluxes, and in the interpretation of the final results.

#### Accumulated Errors

The combined errors in the magnitudes of the quantities are estimated at 1.5 per cent for the relative humidity, 0.15 degrees C for the wet and dry bulb temperatures, 0.2 gm. per kg. for the specific humidity, and 5.0 cm. per sec. for the vertical wind. As stated previously, the errors in the fluxes and spectra depend on the errors in the fluctuations and not those in the magnitudes. These fluctuation errors are estimated to be 0.03 degrees C for the wet bulb, 0.02 degrees C for the temperature, 0.3 per cent for the relative humidity, 0.05 gm. per kg. for the specific humidity, 0.0005 gm. per cubic metre for the density, and 5.0 cm. per sec. for the vertical wind. Under some conditions these errors are smaller.



### Site and Instrument Layout

The trials were conducted on the 1000 square mile test area at Defense Research Establishment Suffield, located in southeastern Alberta, twenty-eight miles northwest of Medicine Hat. Very gently rolling terrain and short grass are typical of the test area.

Two sites were used but the immediate vicinities were very similar. Flat terrain, a few sloughs, and short grass (two to six inches) interspersed with low bushes (one to two feet) are typical of the test sites. Soil moisture was plentiful in the cool wet spring of 1967. The soil moisture became depleted as the weather turned hot and dry about mid-June. Only the first trial was conducted under the spring conditions.

The instruments were installed windward of the tower and instrument van to eliminate interference. The vectorvane and thermocouples at two metres were placed close together in such a way as to minimize interference between them. Grouped near these were the two-metre psychrometer, the one-half-metre wet bulb, the temperature gradient thermocouples, and the two-metre cup anemometer. The tower was instrumented with the four-metre wet bulb and the cup anemometers at eight and sixteen metres. The arrangement, spread out for identification, is shown in Illustration 3.

### Raw Data

The trial lengths varied from twenty to ninety minutes. The steady state mean values were tabulated throughout the period and are given in Appendix A. The voltage signals from the thermocouples and vectorvane were recorded on four channels of the magnetic tape with the thermocouple calibrations before and after the trial. Digitization, conversion, and calculation yielded the temperature, relative humidity, specific humidity, and vertical wind time series to be used in the analysis.



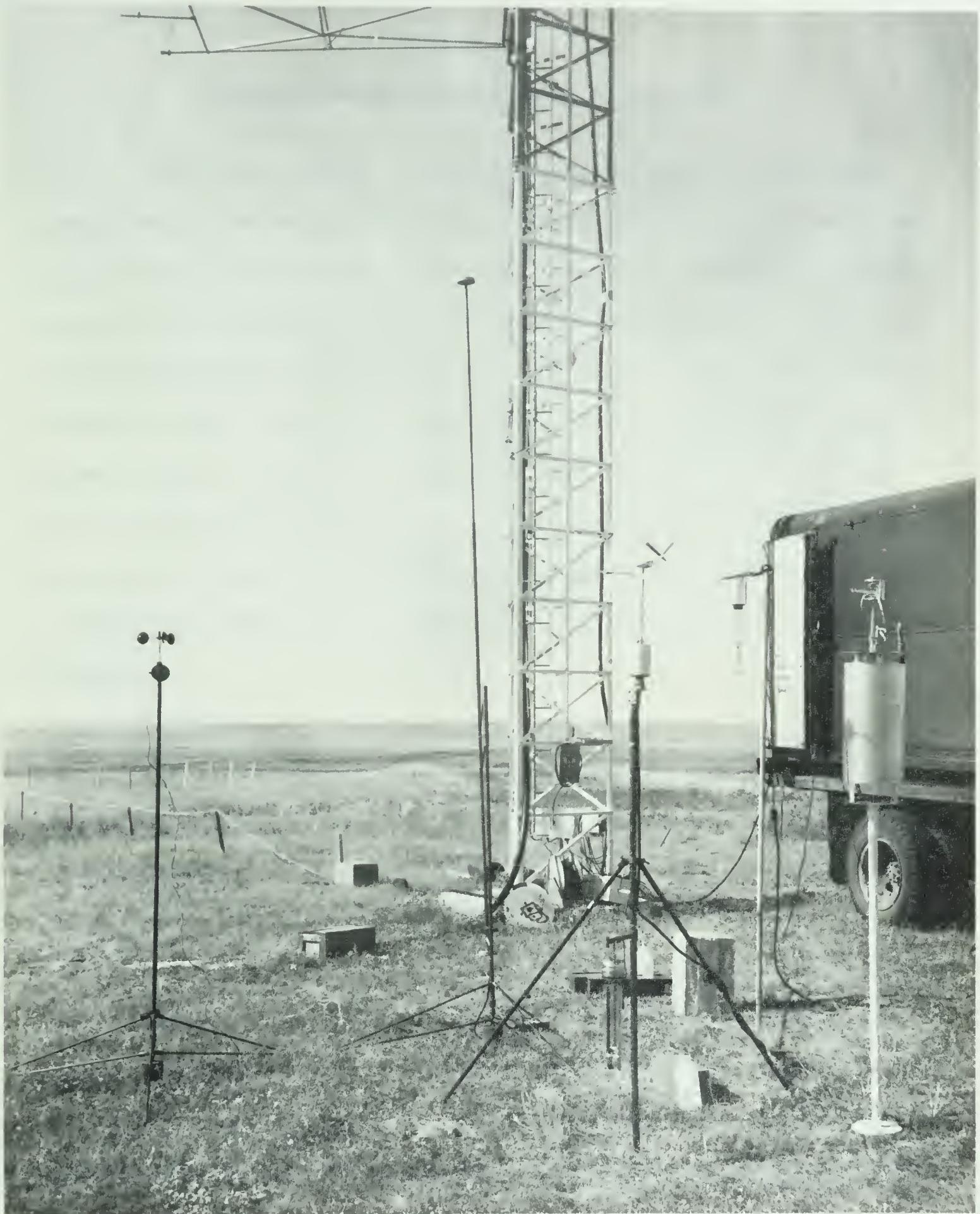


Illustration 3 - Instrument Layout.



## CHAPTER III

### MODIFICATION AND CONVERSION OF THE RAW DATA

The voltage signals of the thermocouples and the vectorvane, described in the previous chapter, were the raw data used to calculate the different time series. These series of data, recorded on four channels of the magnetic tape, were continuous and, therefore, had to be digitized for the computer programs. Modification and conversion to suitable units lead to the temperature, the wet bulb depression, the wind velocity, and the wind elevation angle time series. From these data the wet bulb, the relative humidity, the specific humidity, and the vertical wind component series were calculated. Finally, the linear trends were removed to eliminate errors that would occur in the spectral analysis.

#### The Data Period

The four channels on the magnetic tape were digitized separately at the rate of one count per sec. by averaging the trace over one second. Simultaneously, the paper punch tape assembly punched the value on the paper tape. The values were in the range -500 to +500 units. The digitized data on the paper punch tapes were then plotted and printed by the computer. From the plotted and printed values a suitable ten minute period was chosen<sup>1</sup>. First, the choice was made so that the interval did not include winds less than one metre per sec. Second, spurious values occurred in some trials and, consequently, the period was chosen to minimize the number of these values. Any values of this type that

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<sup>1</sup>Because a change of character occurred at the end of trial #13 only the first 500 seconds were used in the analysis.



remained in the period were replaced manually by representative values or were assigned the average of the two preceding values by the computer program. Third, for neutral trials which were taken during the gradual change from unstable to stable conditions, the period had to be chosen during the interval of near-neutral lapse rate. The chosen ten minute period provided the data for the analysis.

The start of this period was marked on all four channels of the magnetic tape by recording a constant voltage on top of the trial just prior to the start time. At the beginning of the period this voltage was cut off. The effect of this, after digitization, was to have several counts of a constant value just before the period, one number consisting of part constant and part trial, and then the digitized trial values.

By doing the digitization at eight counts per sec. it was possible to have a near-simultaneous start of the four time series. The error involved is estimated to be less than one tenth of a second<sup>2</sup>. This procedure yielded digitized time series that were almost simultaneous.

#### Data Modification

Preliminary analyses of the ten minute periods showed that a longer trial length was necessary to resolve important low frequencies in the time series. Only four of the trials were suitable for periods of 33 1/3 minutes (2000 seconds). Two trials had unstable lapse rates and two stable. Since it was time-consuming to digitize at eight counts per sec. these trials were done at one count per sec. A cor-

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<sup>2</sup>For an example of this procedure see Appendix B.



rection was applied to the four time series of each trial to improve their simultaneousness.

For the shorter trials, a computer program was written which read the paper punch tapes and increased the time interval between data points to one second by averaging over eight values. The final form of the digitized raw data was four time series of 600 values, each having ten minutes of data at one count per sec. For the longer trials, each of the four series consisted of 2000 values with a time increment of one second.

The raw data of temperature and wet bulb depression were modified by removing the linear trends. This was done by using the slope calculated from the average values of the first and last thirds of the data. Then the mean was subtracted from the data. The result was temperature and depression fluctuations about a zero line. Initially, the wind data from the vectorvane were used to calculate the wind speeds. The average speeds were found to be substantially greater than the mean obtained from a small cup anemometer at two metres. To eliminate this error, the mean was subtracted from the wind data leaving fluctuations. Since it was almost impossible to set the vectorvane horizontally, Cramer and Record<sup>3</sup> assumed the mean elevation angle over a set of data was zero. For sufficiently long periods this approximation is fairly accurate. Therefore, the mean was removed from the elevation angle data. The computer program, that did the modification, also converted the data to the desired units

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<sup>3</sup>H.E. Cramer and F.A. Record, Measurements of the Structure of Turbulent Flow at a Height of 2.3 Meters, Massachusetts Institute of Technology, Scientific Report No. 1, South Dartmouth, Mass., 1952, p. 12.



to form the new time series.

### The Time Series

The vertical wind component can now be calculated from these modified data. Wind tunnel test on the MRI vectorvane supplied the conversion factor used to change the wind data to wind fluctuations in cm. per sec. The vectorvane was also calibrated for the elevation angle so that the elevation data could be converted to radians. The cup anemometer wind at two metres, averaged over ten or fifteen minute periods, supplied the base line to which the wind fluctuations were added. The vertical wind component,  $w_i$ , in cm. per sec. was calculated for the ten minute periods by the equation:

$$w_i = (V + C_1 V_i) \sin C_2 A_i \quad i = 1, 2, 3 \dots N \quad (3.1)$$

where  $V$  is the mean two metre wind speed,  $V_i$  the modified wind data,  $A_i$  the elevation angle data,  $N$  the length of trial (600 or 2000), and  $C_1$  and  $C_2$  the conversion factors for the wind and elevation data<sup>4</sup>. The result is a vertical velocity time series of  $N$  values with a time increment of one second.

To convert the modified raw data of temperature and depression into temperature fluctuations in degrees C a calibration constant is required. From the temperature-voltage relationship of the thermocouples, the amplification, and the magnetic tape recorder range the calibration is approximately one degree C for 144 digitized units. Changes in environment and drift in the amplifiers cause this relationship to vary

<sup>4</sup>These conversion factors were 6.209 and 0.00244 respectively.



slightly from trial to trial. Thus, a calibration was done on both the temperature and depression channels before and after each trial. Specific voltages, in steps, were fed to the thermocouple amplifiers and recorded. The voltage steps, each lasting for 18 seconds, corresponded to temperature steps of one degree F from -5 to +5 degrees F. The digitized output of the calibration was averaged over each step and plotted against its corresponding temperature or depression. Straight lines were fitted to these points by the least squares method. The slopes of the two lines were the conversion factors used to change the units to temperature or depression fluctuations in degrees C.

The temperature and wet bulb depression measured at five minute intervals by the psychrometer at two metres supplied the base lines for the fluctuations. In some of the trials, especially the neutral cases, there was a linear trend in the temperature and the depression of the psychrometer readings. These were left in the series until the other series were calculated. Then the linear trends were removed. If an obvious trend existed, the base lines of the temperature and depression were determined from the two-metre psychrometer readings by the least squares method. Otherwise, the temperature and the depression were assumed constant throughout the period. The temperature time series in degrees C,  $T_i$ , was determined by:

$$T_i = T + C_3 T'_i + M_1 i \quad i = 1, 2, 3, \dots, N \quad (3.2)$$

where  $T$  is the mean temperature or the temperature at the start of the trial if a trend existed,  $T'_i$  the modified temperature data,  $C_3$  the conversion factor for the temperature data, and  $M_1$  the linear trend of the



temperature. The wet bulb series in degrees C,  $T_{wi}$ , was determined by:

$$T_{wi} = T_i - D - C_4 D_i - M_2 i \quad i = 1, 2, 3, \dots, N \quad (3.3)$$

where  $D$  is the initial or mean wet bulb depression,  $D_i$  the modified depression data,  $C_4$  the conversion factor for the depression data<sup>5</sup>, and  $M_2$  the linear trend of the depression.

From the temperature and wet bulb time series the vapour pressure was calculated and used to evaluate the relative humidity and specific humidity series. The Goff-Gratch equation<sup>6</sup> was used to calculate saturation vapour pressures.

$$\begin{aligned} \log_{10} e_{Ti} &= 5.028 \log_{10} T - 10.795737 (T-1) \\ &+ .000150475 (1-10^{(8.2969 (1-1/T))}) \\ &- .00042873 (1-10^{(4.76955 (T-1))}) \\ &+ 0.7861406 \quad i = 1, 2, 3, \dots, N \quad (3.4) \end{aligned}$$

The vapour pressure in millibars,  $e_{Ti}$ , is the saturation vapour pressure,  $e_{si}$ , when  $T = 273.16/(273.15 + T_i)$  and the saturation vapour pressure at the wet bulb temperature,  $e_{wi}$ , when  $T = 273.16/(273.15 + Tw_i)$ . The actual vapour pressure in millibars,  $e_i$ , is obtained from the psychometric equation:

<sup>5</sup>The values of the conversion factors for temperature and depression units were both approximately -0.00069. They were negative because a negative digitized value corresponded to a positive temperature fluctuation.

<sup>6</sup>J.E. McDonald, "Saturation Vapour Pressure over Supercooled Water", Journ. of Geophys. Research, Vol. 70, No. 6, 1965, p. 1553.



$$e_i = e_{wi} - (.00066 (1 + .00115 Tw_i)) p (T_i - Tw_i)$$

$$i = 1, 2, 3 \dots N \quad (3.5)$$

where  $p$  is the atmospheric pressure in millibars. The relative humidity time series in per cent,  $RH_i$ , is given by:

$$RH_i = \frac{100 e_i}{e_{si}} \quad i = 1, 2, 3 \dots N \quad (3.6)$$

The specific humidity series in grams per kilogram,  $q_i$ , is calculated from:

$$q_i = \frac{622 e_i}{p} \quad i = 1, 2, 3 \dots N \quad (3.7)$$

where  $p$  is again the atmospheric pressure.

The three time series, temperature, specific humidity, and relative humidity, had their linear trends removed at the end of the computer program. These series and the vertical wind series were printed and plotted. The plots for the nine short trials and four long trials are shown in Appendix C.

Each time series consists of 600 or 2000 values with a one second time interval. The temperature, the specific humidity, and the vertical wind series are used to calculate the heat and moisture fluxes during the short trial. The temperature, the vertical wind, specific humidity, and the relative humidity are used in spectral analysis.



## CHAPTER IV

### BASIC ANALYSIS AND OBJECTIVES

The amplitude and frequency of relative humidity fluctuations are of concern to some scientific fields. An understanding and a knowledge of the relative humidity changes with time could be helpful in determining the effects on insects and microorganisms. Thus, the first objective of this thesis is to determine the characteristics of relative humidity fluctuations and some of the relationships between relative humidity variations and other micrometeorological parameters. This involves spectral analysis of the time series and statistical properties of the sample. For practical reasons, the Eulerian coordinate system (fixed in space) is used in spite of the desirability of the Lagrangian system (following the motion). No attempt to relate the two systems is made. The second objective is the investigation of the vertical transport of heat and moisture by eddies. The coefficients of eddy transport are calculated from the gradients and the fluxes. The effect of atmospheric parameters on the coefficients and their ratio is scrutinized. The method and the accuracy of the flux measurement by the instruments described in Chapter II are examined.

#### Spectral Analysis

Spectral analysis of time series is the basis for determining the major frequencies of the fluctuations. The amplitude of the variations is given by the standard deviation influenced somewhat by the skewness. Estimating and predicting the amplitude and frequency of major relative humidity changes may be done from relationships to other properties. Such relationships are developed from two groups:



- 1) the steady state variables such as stability, relative humidity gradient, wind shear, and wind speed.
- 2) the standard deviation, skewness, and spectra of temperature, specific humidity, and vertical wind.

The first group could be used as predictors with the latter group being useful in diagnosis. The determination of relationships between the parameters of the first group and relative humidity fluctuations involves the effects of the parameters on the spectrum of relative humidity. These effects are best discovered by comparing similar trials. Repeatability of any aspect of the spectra is also important in revealing certain characteristics. For the second group, one theoretical dependency is the inverse relationship between temperature and relative humidity. The convection of moist or dry air modifies the magnitude and, sometimes, even the sense of this relationship. These changes, their causes, and their variations with stability and frequency are examined through spectral analysis. The resulting relationships between the spectra could be useful in prediction. That is, relative humidity fluctuations could be forecast from estimates of temperature and vertical wind fluctuations obtained from steady state variables. These forecasts may be easier than those from direct, but uncertain, correlations between relative humidity and the steady state variables. The spectral relationships would be of diagnostic value also. In the absence of relative humidity measurements its spectra could be estimated from the spectra and statistical properties of the temperature, specific humidity, and the vertical wind. The basis for determining these relationships is the spectral analysis of the quantities involved.



Specifically, the objectives of the relative humidity study by spectral and statistical analysis are:

- 1) to investigate the standard deviation of relative humidity with respect to:
  - a) the vertical gradient of relative humidity,
  - b) the standard deviations of the temperature, specific humidity, and the vertical wind,
  - c) the wind speed and shear,
  - d) the range and the maximum change in one second of the relative humidity,
  - e) the cross covariance of the vertical wind and relative humidity.
- 2) to relate the relative humidity skewness to:
  - a) the temperature, specific humidity, and vertical wind skewnesses,
  - b) the amplitude of relative humidity fluctuations.
- 3) to discover characteristics of the relative humidity spectra and relate these to specific causes:
  - a) frequency of major peaks in the spectra,
  - b) minus five-thirds law of the inertial subrange.
  - c) stability, relative humidity gradient, velocity and sample length,
- 4) to relate the relative humidity spectra to:
  - a) the temperature spectra,
  - b) the specific humidity spectra,
  - c) the vertical wind spectra.



The spectral analysis is based on the summary by McIntosh<sup>1</sup> of techniques developed by Blackman and Tukey<sup>2</sup> and Jenkins<sup>3</sup>. The computational formulae for discrete data are given in Appendix D.

The spectral estimates are calculated from the three time series: temperature, relative humidity, and vertical wind. The frequency range of the spectra is bounded by the length of the trial and the time constant, T, of the instrument. To measure the high frequency fluctuations accurately, the time increment of the data, t, must be larger than  $2.5T$ . A time constant of 0.4 seconds limits the time increment to one second.

The highest frequency in the spectral analysis, called the Nyquist or folding frequency, is equal to  $1/2t$  (0.50 cycles per sec. for these trials). The lowest frequency in the analysis is determined by the maximum lag, M, used in calculating the covariance. This maximum lag must not exceed about one tenth of the number of data points. Because the length of the short trials was 600 seconds the maximum lag used was fifty. The long trials were 2000 seconds long so a maximum lag of 200 was used. The lowest spectral frequency is given by  $1/2Mt$ . The resolution or frequency difference between consecutive spectral estimates is also given by  $1/2Mt$ . Thus, for the short trials, the spectral

<sup>1</sup>T. McIntosh, Spectral Analysis, unpublished paper, Defense Research Establishment Suffield, Ralston, Alberta, 1967, pp. 1-14. References for this paper are marked with a \* in the bibliography.

<sup>2</sup>R.B. Blackman and J.W. Tukey, Measurement of Power Spectra, Dover Publications, New York, 1958, pp. 1-190.

<sup>3</sup>G.M. Jenkins, "General Considerations in the Analysis of Spectra", Technometrics, Vol. 3, No. 2, 1961, pp. 133-166, and "A Survey of Spectral Analysis", Applied Statistics, Vol. 14, No. 1, 1965, pp. 2-32.



frequency range is 0.01 to 0.50 cycles per sec. with a resolution of 0.01 cycles per sec. For the long trials, the range is 0.0025 to 0.5000 cycles per sec. with a resolution of 0.0025 cycles per sec.

The spectral estimates obtained from the data can be improved by several methods. The three used in this analysis are prewhitening, averaging correction, and smoothing of the covariances.

To minimize the problems of noise and distortion in the time series, prewhitening of the series is performed by the formula:

$$x_i' = x_i' - b x_{i-1}' \quad i = 1, 2, 3, \dots, N \quad (4.1)$$

where  $b$  is equal to 0.75,  $N$  is 600 or 2000, and the primed  $x$ 's the original series. This process emphasizes the high frequency fluctuations relative to the low frequencies. The spectral estimates,  $U_n$ , are then recovered from the prewhitened estimates,  $V_n$ , by the transformation:

$$U_n = \frac{V_n}{1 + b^2 - 2b \cos \frac{\pi n}{M}} \quad n = 0, 1, 2, \dots, M \quad (4.2)$$

When the data were digitized by averaging over one second, an error is introduced into the spectral estimates. This error tends to reduce the high frequency estimates. The corrected spectral estimates,  $S_n$ , are obtained from the previous estimates by:

$$S_n = U_n \frac{(\pi n/2M)^2}{(\sin \pi n/2M)^2} \quad n = 0, 1, 2, \dots, M \quad (4.3)$$



In the calculation of the spectral estimates from the covariances smoothing is done automatically in the formulae. Only the hanning type of smoothing is used.

The spectral estimates are also normalized by dividing them by the covariance at zero lag. Because spectral analysis is the process of dividing the variance into frequency bands the normalized estimates give the percentage of the variance in a specified band. That is, the area under the normalized spectra plotted against frequency is theoretically equal to unity. In practice the area is slightly less than unity.

Errors in the spectra come primarily from aliasing. This arises when the true spectrum has variance at frequencies above the Nyquist frequency. Because the instruments and the sampling are incapable of measuring these frequencies, their energy appears at lower frequencies. The source of this error and its effect are illustrated by Lumley and Panofsky<sup>4</sup>. The aliasing error in the spectral estimates increases with frequency. For this reason the estimates at the higher frequencies must be used with caution.

A Fortran computer program was written by the author to calculate the spectral estimates. The program was tested with dummy data to determine its accuracy. Small errors were detected and the necessary adjustments were made. The computations of the program were checked by using data which had known amplitudes and frequencies. The spectra were consistent with the data and justified the use of at least three significant

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<sup>4</sup>J.L. Lumley and H.A. Panofsky, The Structure of Atmospheric Turbulence, John Wiley & Sons, New York, 1964, pp. 54-56.



digits in the output.

### Flux Calculations

The vertical transport of heat and moisture by eddies in the lower levels of the atmosphere can be calculated from measurements made by fast-response instruments. The parameters required are temperature, specific humidity (or water vapour pressure), and vertical wind. The data available in the nine short trials are well suited for the calculation of vertical flux transport. The flux values and the vertical gradients are used to get coefficients of heat and moisture transfer, often called eddy conductivity and eddy diffusivity. The coefficients and their ratio are examined to determine the effect, if any, of parameters such as stability, wind speed, wind shear, standard deviation, and skewness.

Specifically, the objectives of the flux and eddy coefficient investigation are:

- 1) to calculate:
  - a) the heat and moisture fluxes,
  - b) the coefficients of heat and moisture transfer,
  - c) the ratio of the coefficients of eddy transfer.
- 2) to detect any variation in the coefficients or their ratio with respect to:
  - a) stability,
  - b) wind speed and shear,
  - c) standard deviations and skewness.
- 3) to estimate the errors in the fluxes and the coefficients caused by:
  - a) slow response of the instruments,



- b) wind speed,
- c) inaccurate measurement of the humidity gradient.

Calculation of the eddy transports is done using the method developed by Swinbank<sup>5</sup>. The results are due to eddy transport only. Any transport caused by a net vertical mass transport of air is eliminated. The vertical heat flux by eddies,  $F_h$ , in cal. per sq. cm. per sec. is obtained from:

$$F_h = c_p \overline{(dw)}_i' T_i' \quad i = 1, 2, 3, \dots, N \quad (4.4)$$

where  $c_p$  is the specific heat of air at constant pressure,  $d$  is the air density,  $w$  the vertical wind, and  $T$  the air temperature. The prime denotes fluctuations about the mean value, and the bar denotes average.

A slight variation and discussion of Equation (4.4) is given by Businger and Miyake<sup>6</sup>. The flux is related to spectral analysis because the bar and the two variables beneath it are identical to the cross covariance of the variables at lag zero. (Equation (D.4) in Appendix D). The vertical moisture flux by eddies,  $F_w$ , in g. per sq. cm. per sec. is calculated from:

$$F_w = \frac{.622}{p} \overline{(dw)}_i' e_i' \quad i = 1, 2, 3, \dots, N \quad (4.5)$$

<sup>5</sup>W.C. Swinbank, "The Measurement of Vertical Transfer of Heat and Water Vapour by Eddies in the Lower Atmosphere", Journ. of Met. Vol. 8, No. 3, 1951, pp. 135-145.

<sup>6</sup>J.A. Businger and M. Miyake, "Justification for Neglecting the Third Moment in the Steady State Expression for the Turbulent Heat-Flux Near the Ground", Quart. Journ. of Roy. Met. Soc., Vol. 94, 1968, pp. 206-207.



where  $p$  is the atmospheric pressure and  $e$  the water vapour pressure.

Again the bar and the two variables are equal to the cross covariance at lag zero.

The fluxes are also related to the vertical gradients. Using the flux and vertical gradient values, the coefficients of eddy transfer are obtained. The coefficient of eddy heat transfer, or eddy conductivity,  $K_h$ , is calculated from the equation:

$$F_h = - K_h c_p d (r - r_d) \quad (4.6)$$

where  $r_d$  is the dry adiabatic gradient and  $r$  the actual vertical gradient. Correspondingly, the coefficient of eddy moisture transfer, or eddy diffusivity,  $K_w$ , comes from:

$$F_w = - K_w dr_q \quad (4.7)$$

where  $r_q$  is the vertical gradient of the specific humidity.

The computer program used in calculating the spectral estimates contained a section which determined the values of the heat and moisture fluxes for each short trial.



## CHAPTER V

### RELATIVE HUMIDITY THEORY

Relative humidity fluctuations at a fixed point in space depend on the spatial distribution of relative humidity, the growth and decay of the distribution with time, and the advection of the distribution past the sensors. Because the flow of air is turbulent, the relative humidity fluctuations are related to some of the properties of turbulence. The study of the fluctuations consists of two parts: amplitude and frequency. The standard deviation is the main basis used to investigate the amplitude. Spectral analysis, in normalized form, is used for the frequency portion. To simplify the problem, horizontal homogeneity and stationarity of the mean values are assumed for the duration of each trial.

#### Standard Deviation

With the above assumptions, changes in relative humidity are caused by the turbulent eddies passing the sensors. The changes can also be looked upon as the vertical motion of air parcels that originate at different heights. The magnitude of the vertical relative humidity gradient and the displacement determine the amplitude of the changes. Accordingly, the variations of relative humidity are a function of the product of the gradient and the vertical motion. Using the standard deviation,  $s$ , as a measure of the variations then:

$$s_{RH} = a s_w r_{RH} \quad (5.1)$$

where "a" is a variable with dimensions of time,  $r$  is the gradient, subscript RH is relative humidity, and subscript w is vertical wind.



The value of "a" is a measure of the duration of an eddy. For this reason its value will change for different turbulent conditions. The main influence is the wind speed because it determines the length of time one eddy will effect the sensors. The wind speed and the size of the eddies are interrelated. These two factors act in opposite senses and make the wind effect on the value of "a" hard to determine.

The vertical gradient of the relative humidity can also be used in another approach to determine the standard deviation. The range of relative humidity is equal to the vertical gradient times the maximum vertical displacement. This is represented by the equation:

$$R_{RH} = d |r_{RH}| \quad (5.2)$$

where R is range and d the maximum displacement.

For normal distribution, 99.7 per cent of a random sample is within three standard deviations of the mean. This yields the approximate relations:

$$R_{RH} = 6.0s_{RH} \quad (5.3)$$

and

$$R_w = 6.0s_w \quad (5.4)$$

The maximum displacement is estimated by the sum of the maximums of updraft and downdraft distances. Converting these to velocities and time the equation becomes:

$$d = 0.5R_w t \quad (5.5)$$

where t is the duration of the major eddy.



Using the logarithmic wind profile and a relationship between the friction velocity and standard deviation of vertical wind, Lumley and Panofsky<sup>1</sup> obtained:

$$s_w = \frac{0.4 AV}{\ln(z/z^*)} \quad (5.6)$$

where A is a constant equal to 1.05, z is the height (2m.) of the mean wind speed, V, and  $z^*$  is the roughness length. Deacon<sup>2</sup> found the roughness length to be 0.7 cm. at Suffield. For high wind speeds, Hage<sup>3</sup> calculated similar values but as the wind decreased  $z^*$  increased. Using 0.7 cm. for the roughness length and Equation (5.4), Equation (5.6) becomes:

$$R_w = 0.45V \quad (5.7)$$

For slow winds and a roughness length of 7.0 cm. the above constant is 0.75.

Combining Equations (5.2), (5.3), (5.5) and (5.7) the final result is:

$$s_{RH} = bVt |r_{RH}| \quad (5.8)$$

with b having a theoretical value of 0.037 when A is 1.05 and  $z^*$  is 0.7

<sup>1</sup>J.L. Lumley and H.A. Panofsky, The Structure of Atmospheric Turbulence, John Wiley & Sons, New York, 1964, p. 136.

<sup>2</sup>E.L. Deacon, "Vertical Diffusion in the Lowest Layers of the Atmosphere", Quart. Journ. of Roy. Met. Soc., Vol. 75, 1949, pp. 100-101.

<sup>3</sup>K.D. Hage, "On the Dispersion of Large Particles from a 15-M Source in the Atmosphere", Journ. of Met., Vol. 18, No. 4, 1961, p. 536.



cm. For  $z^*$  equal to 7.0 cm.  $b$  increases to 0.052. The duration of the major eddy,  $t$ , is obtained from the vertical wind spectrum. It is equal to the inverse of the frequency of the second maximum in the spectra. The first maximum is not used because it is not caused by eddies but by horizontal inhomogeneity.

Equation (5.8) is an improvement on (5.1) because the constant has a theoretical basis. However, accurate values of  $t$  and  $s_w$  can be obtained only from actual measurements. Equation (5.6) can not be used for  $s_w$  in (5.1) because this essentially reduces it to the form of (5.8). The eddy duration,  $t$ , should be a constant under similar turbulent conditions, but varies with stability, wind, and other factors. If the three variables in (5.8) can be predicted then a forecast value of the relative humidity variance is available.

Using similarity theory, Lumley and Panofsky<sup>4</sup> derived the following formula for the variance of temperature;

$$s_T = CT^*S \quad (5.9)$$

stating,

"It is quite likely that variances of other scalars will behave as indicated ..., with  $T^*$  replaced by the appropriate dimensional quantity, which is independent of height and depends on the gradient of the scalar in question".

In Equation (5.9),  $S$  is the non-dimensional wind shear,  $(kz/u^*) (dV/dz)$ , and  $C$  is a constant if the ratio of eddy transfer coefficients of

<sup>4</sup>Lumley and Panofsky, op. cit., pp. 158-159.



temperature and momentum is a constant.

Similar to the temperature scale  $T^*$ , a relative humidity scale  $RH^*$  is defined as:

$$RH^* = \frac{1}{ku^*} \left| \overline{wRH'} \right| \quad (5.10)$$

where  $k$  is the von Kármán constant (0.4),  $u^*$  is the friction velocity, the prime denotes fluctuations, and the bar denotes average or cross covariance. The friction velocity is obtained from<sup>5</sup>:

$$s_w = Au^* \quad (5.11)$$

where  $A$  is the same constant as in Equation (5.6).

The relative humidity equivalent of (5.9) is given by:

$$s_{RH} = CRH^*S \quad (5.12)$$

where  $S$  is the same as above and  $C$  depends on the ratio of eddy transfer coefficient of relative humidity and momentum. Assuming  $C$  is unity, the variance equation becomes:

$$s_{RH} = c \frac{dV}{dz} \frac{\left| \overline{wRH'} \right|}{s_w^2} \quad (5.13)$$

where  $c$  is  $zA^2$  and for a measurement height of two metres is theoretically equal to 2.2. The vertical wind standard deviation,  $s_w$ , may be obtained from (5.6). The wind shear,  $dV/dz$ , is calculated from the two

<sup>5</sup>Lumley and Panofsky, op. cit., p. 134.



and eight metre anemometers. The shear changes with stability and mean wind speed. However, the cross covariance is difficult to determine without actual measurements and, therefore, renders this equation less useful for prediction.

A statistical approach to relate relative humidity variations to changes in other variables consists of a linear combination of those variables. The relative humidity of a given air sample depends on the temperature and the specific humidity of that air sample. Any variation in either causes a related change in relative humidity. For turbulent flow, the vertical motion of air parcels is a third factor influencing the relative humidity at a fixed point. Interactions among the variables may cause distortion of the coefficients. Assuming a linear combination and using the standard deviations as a measure of the fluctuation, the equation is:

$$s_{RH} = a_T s_T + a_q s_q + a_w s_w \quad (5.14)$$

The coefficients,  $a$ 's are certainly functions of stability and should only be calculated from similar trials.

The relation between the standard deviation of the relative humidity and its maximum change in one second is purely statistical. The ratio of these two quantities is taken to determine if it is a constant.

The four equations for determining the relative humidity standard deviation can all be used in diagnostics with Equation (5.8) being the only suitable one for prediction.



### Skewness

Skewness is a measure of the distortion in a frequency distribution. Positive skewness indicates a larger deviation to the positive side of the mean than to the negative side. It also indicates that less than half of the values are greater than the mean. Consequently, positive skewness is interpreted as fewer values to the positive side of the mean but having larger deviations from the mean. On the negative side, there are more values but they are grouped closer to the mean. Negative skewness is simply the reverse.

The relative humidity skewness has an effect on the amplitude of the fluctuations. The modification is usually small but is significant for a large skewness. Using both the skewness and the standard deviation, the amplitude of variations are more accurately described. For this reason, a measure of the magnitude of the skewness and its sign are useful.

A comparison of the skewness signs for relative humidity, temperature, specific humidity, and vertical wind is made. Any correlations of the signs is purely statistical with no theoretical basis given. The effect of stability on skewness sign is also investigated.

The maximum and minimum fluctuations are compared with the sign of the skewness. This illustrates the effect of skewness on the fluctuations.

Linear combination of the four skewnesses, k's, determines the coefficients, b's, for the equation:

$$k_{RH} = b_T k_T + b_q k_q + b_w k_w \quad (5.15)$$



Again the coefficients depend on stability and are determined only for similar trials. Interactions, as before, modify the coefficients.

Using the above equation as an estimate of the magnitude, and the sign of the relative humidity skewness, the effect on the amplitude of relative humidity changes is determinable.

#### Spectral Characteristics

The relative humidity spectra are studied from the graphs of normalized variance versus non-dimensional frequency. The form used is similar to that used by Lumley and Panofsky<sup>6</sup>. The logarithm of the frequency is used as the abscissa and the normalized variance as the ordinate. To normalize the frequency,  $n$ , in cycles per sec. to a non-dimensional frequency,  $f$ , the relationship:

$$f = \frac{nZ}{V} \quad (5.16)$$

is used. This corrects for shifts in spectra caused by different heights of observations and different wind speeds. For comparison purposes, the area beneath the spectral curve is equated to unity. This is achieved by using:

$$\sum (S^*/s^2)dn = 2.303 \sum (nS^*/s^2)d(\log_{10}f) = 1.0 \quad (5.17)$$

The summation is over all frequencies,  $n$ ,  $S^*$  is the spectral estimate at  $n$ , and  $s^2$  is the total variance. Smoothing and corrections cause the actual area to vary from this value. Spectral graphs are plotted with

<sup>6</sup> Ibid., p. 170



$2.303 \text{ nS}^*/\text{s}^2$  versus  $\log_{10} f$ . These also contain a smoothed spectral plot.

The characteristics of the spectra are described in terms of the normalized variance,  $S = 2.303 \text{ nS}^*/\text{s}^2$ , and  $f$ , the non-dimensional frequency.

The general shape of the relative humidity spectra is expected to be bell-shaped, similar to those of temperature and vertical wind. Sharp maxima that occur need to be tested for significance. This is done by calculating an envelope of confidence limits on each side of the mean spectra. The approximate confidence interval for a spectrum, is obtained using the values<sup>7</sup>:

$$\exp(\ln S^*(f) \pm z_a \sqrt{kM/N}) \quad (5.18)$$

where  $k$  is the number of degrees of freedom and  $z_a$  is the upper  $a/2$  per cent limit of the normal distribution. This approach is inferior to the comparison of spectra obtained under similar conditions. As Jenkins<sup>8</sup> states,

"We are of the opinion that confidence intervals for single spectra are not very important nor useful since in addition to the three assumptions listed above, they also depend heavily on stationary and normality assumptions. It is usually far more important to see that when the experiment is repeated a spectrum is obtained which bears a reasonable resemblance to the first".

In the comparison of the trials only those with the same stability are used. The normalization of the spectra makes this comparison easier.

<sup>7</sup>G.M. Jenkins, "General Considerations in the Analysis of Spectra", Technometrics, Vol. 3, No. 2, 1961, p. 163.

<sup>8</sup>Loc. cit.



When a certain maximum reoccurs at the same frequency in several trials confidence in its reliability and importance increases.

The spectrum of an atmospheric scalar is affected by stability, vertical gradient, wind speed, and sample length. It has been shown that the temperature and vertical velocity spectra shift to lower frequencies as stability decrease. This is caused by convection which occurs at lower frequencies than mechanical turbulence. For these reasons the relative humidity spectra should show a similar shift. Any effect of advection due to the mean wind is eliminated by the normalization of the frequency. The wind speed, combined with the vertical gradient, does affect the type of relative humidity fluctuations. When both of these are large, the variations caused by turbulence should overpower horizontal inhomogeneity and the spectrum shifts to high frequencies. A shift to lower frequencies should occur with light winds and small gradients.

The effect of sample length can be examined by comparing the four trials which had short and long samples. Changes in the maximum lag and in the sample length of the long trials is used to determine any effects.

For scalar fluctuations there is evidence that their spectra obey a minus five-thirds law<sup>9</sup>. This inertial subrange is bound on the low frequency side by the reciprocal of the observation height and on the high side by 1000 cycles per m. It has been shown that the minus five-thirds law holds at frequencies much lower than stated above<sup>10</sup>. Assuming the

<sup>9</sup>Lumley and Panofsky, op. cit., p. 85 and p. 164.

<sup>10</sup>Lumley and Panofsky, op. cit., p. 163.



relative humidity spectra calculated for the nine trials lies in a region of the minus five-thirds law, the relation is:

$$S' = cf^{-5/3} \quad (5.19)$$

where  $c$  is a constant that varies with stability, and  $S'$  is the true spectrum. When the logarithms of the spectrum and frequency are plotted the slope of the line is minus five-thirds. Slopes for the nine short trials are calculated.

Characteristics of the relative humidity spectra reveal the range of frequencies or wave lengths that are most important in the variation of relative humidity. Modification and shift of the spectrum, caused by changes in the steady-state parameters, are useful when a prediction or an estimation of the spectrum is required.

#### Spectral Relations

Relative humidity is a function of temperature and specific humidity. If either is constant, the relative humidity is dependent on one variable only. For neutral stability the normalized spectra of relative humidity and specific humidity are comparable. Similarly, for zero specific humidity gradient the normalized spectra of temperature and relative humidity are coincident. With vertical gradients of both variables, the fluctuations of temperature and specific humidity compete for the control of the relative humidity.

Jenkins<sup>11</sup> used the logarithmic transformation of the spectral

<sup>11</sup>Jenkins, op. cit., pp. 163-164



estimates to compare two spectra of the same variable. The test consists of comparing:

$$D = \frac{1}{M \sqrt{M(k_1 + k_2) / N}} \sum \ln S_1^*/S_2^* \quad (5.20)$$

with the normal distribution. The summation is over all the spectral estimates, and  $k$  is degrees of freedom. To compare the spectra of two different variables,  $S^*$  must be the spectral estimate divided by the variance. For sufficiently small  $D$  the two spectra are not significantly different. By comparing certain frequency ranges the test is used to determine if temperature or specific humidity controls the relative humidity. If so, it is easier to estimate the magnitude and the importance of relative humidity fluctuations in that frequency range.

#### Summary

The amplitude of relative humidity fluctuations can be determined from the standard deviation and skewness or from steady-state parameters. The spectrum can be estimated from steady-state parameters or its relation to other spectra. Combining the spectrum and the amplitude, the variance of the relative humidity can be divided into given frequency ranges, with an estimation of their magnitude and importance.



## CHAPTER VI

### RELATIVE HUMIDITY ANALYSIS

The nine short and four long trials were analyzed by computer programs to obtain the required data for the formulae derived in the previous chapter. Certain errors and problems were discovered. The most serious, as could be expected, is the measurement of the relative humidity gradient by the psychrometers. The accuracy of the thermometers is not sufficient because the slightest error causes a large error in the gradient. Consequently, the gradient used in the analysis is estimated from such items as: normal conditions, the psychrometer readings, the temperature gradient, the standard deviations of relative and specific humidity, temperature, and vertical wind, the phase lead and coherences of temperature, specific humidity, and relative humidity over vertical wind, and the traces of the data. Secondly, the relative humidity and vertical wind data for the long trials were derived from raw data that had been smoothed to a greater extent than the short trials. This reduces the importance of high frequency fluctuations and produces errors in the long trial analysis. Also, the slow response of the vector-vane in light winds causes an underestimation of the vertical wind fluctuations. Keeping these factors and their possible effects in mind, the results of the analysis are used to test the developed theories and equations.

#### Standard Deviation

The constant,  $a$ , of Equation (5.1) is shown in Table I. A test of extreme values rejects 37.23 at the five per cent level. Because #10, #10L and #13 have slow wind speeds and, therefore, underestimation of the



Table I - Values of  $a$  for Equation (5.1)

No.	Stab. <sup>1</sup>	$s_{RH} (\%)$	$s_w (\text{m/sec})$	$r_{RH} (\%/\text{m})$	$a (\text{sec})$
1	u	2.07	0.471	0.491	8.94
8	u	1.28	0.437	0.204	14.36
11	u	1.18	0.248	0.703	6.77
3	n	1.00	0.276	0.411	8.81
6	n	0.429	0.464	0.107	8.61
9	n	1.14	0.298	0.400	9.56
7	s	0.742	0.378	0.317	6.19
10	s	0.986	0.0503*	1.10	17.87
13	s	1.10	0.0466*	0.634	37.23
1L**	u	1.41	0.400	0.491	7.18
8L	u	1.42	0.420	0.204	16.57
7L	s	0.587	0.430	0.317	4.31
10L	s	1.04	0.0534*	1.10	17.88

<sup>1</sup>u is unstable, n is neutral, and s is stable.

\* accuracy is low because the values are small.

\*\*L represents long trials.

vertical wind standard deviation, the value of the constant is too high.

Rejecting these three values, the average of the short trials is 9.0, the long trials 9.4, and combined 9.1. The long trial values are suspect because their values have a large variation and their relative humidity and vertical wind data have systematic errors. No wind or stability effect is discernible so the value of  $a$  is most likely a constant approximately equal to 9.0.

Checking the validity of Equations (5.3) and (5.4) the average factors for the nine short trials are 6.0 and 6.9 respectively. These compare favourably with the given theoretical values of 6.0. The long trials values are larger, as expected. These affect the values of subsequent constants. Equation (5.6) yields standard deviations that are too large. The most likely cause of this is the undermeasurement of



the vertical wind due to response and smoothing. It is also possible that the values given to A and  $z^*$  are in error. The error in this and the previous equation combine and cause an error in Equation (5.7). For wind speeds greater than 3 m. per sec. the average constant is 0.39 compared to the theoretical 0.45. For winds less than 2 m. per sec. the average constant is 0.21. This is in conflict with the theoretical value of 0.75. The answer lies in the lack of response of the vector-vane for slow wind speeds and a change in the character of the flow under these conditions. For these reasons the 0.75 value is the most reasonable.

The above constants affect the constant, b, of Equation (5.8). The calculated values are given in Table II. Also included are the vertical wind spectra values of t and the relative humidity gradients calculated from the theoretical constant, b. There is a general decrease

Table II - Constant b of Equation (5.8)

No.	Stab.	V(m/sec)	t(sec)	$s_{RH}(\%)$	$r_{RH}(\%/\text{m})$	b	$r'_{RH}(\%/\text{m})$
1	u	7.99	16.7	2.07	0.491	0.0316	0.419
8	u	9.31	20.0	1.28	0.204	0.0337	0.186
11	u	3.80	14.3	1.18	0.703	0.0309	0.587
3	n	4.29	14.3	1.00	0.411	0.0396	0.441
6	n	8.15	16.7	0.429	0.107	0.0293	0.085
9	n	6.20	12.5	1.14	0.400	0.0368	0.398
7	s	7.12	11.1	0.742	0.317	0.0296	0.254
10	s	1.65	11.1	0.986	1.10	0.0491	1.04
13	s	1.48	20.0	1.10	0.634	0.0586	0.715
1L	u	6.93	14.8	1.41	0.491	0.0280	0.371
8L	u	8.85	20.0	1.42	0.204	0.0393	0.217
7L	s	6.93	10.8	0.587	0.317	0.0247	0.212
10L	s	1.71	10.8	1.04	1.10	0.0512	1.08

<sup>1</sup> calculated gradients for b = 0.037 except for #10, #10L and #13 where b = 0.052.



of  $t$  with stability as shown by the average of 17.0 for unstable and 14.5 for neutral. Stable conditions show two values with 11.1 the more likely because this is the third maximum in the spectra for #13. However, under certain conditions  $t$  is equal to 20.0.

The constant,  $b$ , is dependent on wind speed. The seven short trials with larger wind speeds have an average value of  $b$  equal to 0.033. For the two slow wind cases  $b$  averages out to 0.054. The long trials have values of 0.030 and 0.051. The average values of the constant are very close to the theoretical values of 0.037 and 0.052 and, therefore, can be considered reliable.

The similarity equation, (5.13), for the relative humidity standard deviation contains a constant,  $c$ , which is proportional to the height. This constant is listed in Table III for the short and long trials. Because no wind measurements were taken below two metres, the one metre

Table III - Constant  $c$  of the Similarity Equation, (5.13)

No.	Stab.	$dV/dz$ (m/sec/m)	$s_{RH}$ (%)	$s_w$ (m/sec)	$\overline{wRH^t}$ (m/sec)	$c$ (m)
1	u	0.575	2.07	0.471	-0.359	2.22
8	u	0.939	1.28	0.437	-0.119	2.19
11	u	0.383	1.18	0.248	-0.0783	2.42
3	n	0.511	1.00	0.276	0.0905	1.65
6	n	0.805	0.439	0.464	-0.0189*	0.62
9	n	0.613	1.14	0.298	0.0715	2.31
7	s	0.798	0.742	0.378	0.100	1.33
10	s	0.466	0.986	0.0503*	0.0108*	0.50
13	s	0.307	1.10	0.0466*	0.00199*	3.91
1L	u	0.575	1.41	0.400	-0.0796	4.93
8L	u	0.939	1.42	0.420	0.0697	3.83
7L	s	0.798	0.587	0.430	0.0429	3.17
10L	s	0.466	1.04	0.0534*	-0.00157*	4.05

\* accuracy is low because the values are small.



wind is assumed to equal one half the two metre wind. Using this value and the eight metre wind, the wind shear,  $dV/dz$ , is calculated. When the covariances are small the errors in the data lead to large errors in the covariances. The values of #6, #10, and #13 are rejected and the short trial average becomes 2.0. The average long trial value excluding #10L, is 4.0. The error here is most likely due to the errors caused by the extra smoothing of the raw data. Therefore, considering the calculation of the wind shear, the constant 2.0 agrees well with 2.2 given by the theory.

The linear coefficients of the standard deviations for the three stability cases are given in Table IV. The coefficients show no logical variation with stability. Often they are large with opposite signs.

Table IV - Linear Coefficients of Standard Deviations

No.	Stab.	$s_{RH}(\%)$	$s_T(^{\circ}\text{C})$	$s_q(2/\text{ks})$	$s_w(\text{m/sec})$	$a_T$	$a_q$	$a_w$
1	u	2.07	0.776	0.202	0.471			
8	u	1.28	0.745	0.152	0.437	-2.79	16.3	2.01
11	u	1.18	0.812	0.181	0.248			
3	n	1.00	0.118	0.225	0.276			
6	n	0.429	0.0809	0.141	0.464	-10.4	10.4	0.429
9	n	1.14	0.0471	0.168	0.298			
7	s	0.742	0.175	0.0439	0.378			
10	s	0.986	0.145	0.0581	0.0503	14.8	-17.5	2.86
13	s	1.10	0.254	0.144	0.0466			

Furthermore, they are large when that particular variable should have little effect. This is especially true in the temperature coefficient of the neutral trials. These characteristics discredit linear combinations and, therefore, the coefficients are expected to be unstable and



of little practical use.

The ratio of the relative humidity standard deviation and its maximum change in one second is listed in Table V. The average ratio for the nine short trials is 4.0 but there is a slight variation with stability. The ratio is largest for neutral trials during which the relative humidity is affected only by the specific humidity. The ratio is less for unstable and stable conditions in which both temperature and specific humidity compete for the control of relative humidity. The average ratios are 0.38, 0.47, and 0.35 for increasing stability.

Table V - Ratio of Standard Deviation and Maximum One Second Change

No.	Stab.	$s_{RH} (\%)$	F* (%)	$s_{RH}/F$	MEANS
1	u	2.07	5.51	0.376	
8	u	1.28	3.30	0.388	0.380
11	u	1.18	3.13	0.377	
3	n	1.00	2.03	0.493	
6	n	0.429	1.02	0.421	0.473
9	n	1.14	2.26	0.504	
7	s	0.742	2.33	0.318	
10	s	0.986	2.98	0.331	0.347
13	s	1.10	2.81	0.391	

\*F is maximum change in one second.

#### Skewness

The four skewnesses for each of the nine short trials are given in Table VI. In eight out of nine cases the signs of the relative and specific humidity agree. Trial #8L has a positive relative humidity skewness. This suggests that the sign of the relative humidity skewness of trial #8 is not representative and should be positive. This change makes the signs agree completely. It also fits the sign convention of positive



relative humidity skewness for unstable and neutral conditions and negative relative humidity skewness for stable conditions. These two relations make a prediction of the skewness sign possible.

Also listed in Table VI are the linear coefficients for each stability case. Similar to the standard deviations, the skewness coefficients are discredited by the lack of logical variation or reasonable consistency. Therefore, they are expected to be unstable and of little practical importance.

Table VI - Skewnesses: Sign Relation and Linear Coefficients

No.	Stab.	$k_{RH}$	$k_T$	$k_q$	$k_w$	$b_T$	$b_q$	$b_w$
1	u	0.255	0.256	0.585	0.247			
8	u	-0.0914	0.196	0.710	0.240	4.07	-0.873	-1.11
11	u	0.0156	0.354	1.53	0.0809			
3	n	0.120	0.818	0.175	0.235			
6	n	0.175	-0.675	0.0818	0.457	-0.0807	0.934	0.0964
9	n	0.635	0.268	0.697	0.0583			
7	s	-0.0572	0.0853	-0.104	0.260			
10	s	-0.0282	0.103	-0.0792	0.0562	-1.29	-1.65	-0.455
13	s	-0.519	0.530	-0.350	0.902			

Table VII illustrates the effect of the skewness sign on the fluctuations. For a negative skewness the maximum departure from the mean is greatest on the low side of the mean. For a positive sign it is on the high side of the mean. This is shown by comparing the signs of the skewness and the sum of the extreme departures. The signs are the same for ten of the thirteen trials and, thus, agree with what is expected.



Table VII - Skewness Sign and Extreme Departures from the Mean

No.	Stab.	$D_{\max}^*$	$D_{\min}^*$	$D_{\max} + D_{\min}$	$k_{RH}$
1	u	6.76	-5.50	1.26	0.255
8	u	3.78	-3.37	0.41	-0.0914
11	u	4.43	-3.50	0.93	0.0156
3	n	2.72	-3.78	-1.06	0.120
6	n	1.27	-1.19	0.08	0.175
9	n	4.64	-2.53	2.11	0.635
7	s	1.79	-2.43	-0.64	-0.0572
10	s	2.51	-2.58	-0.07	-0.0282
13	s	2.73	-4.22	-1.49	-0.519
1L	u	7.45	-4.18	3.27	0.473
8L	u	5.58	-4.99	0.59	0.435
7L	s	2.33	-1.97	0.36	-0.156
10L	s	2.89	-3.45	-0.56	-0.152

\*  $D_{\max}$  and  $D_{\min}$  are maximum and minimum departures from the mean.

#### Spectral Characteristics

The normalized relative humidity spectra were calculated for the thirteen trials. Most of these are given in Appendix E. For #7, #8, and #10, initial observations showed a change in shape from the short to the long trials. After elimination of other possibilities, this is attributed to the extra smoothing the long trials received before the calculation of the relative humidity time series. This change in shape makes any comparison between short and long trials meaningless. A more serious problem is aliasing. Most of the relative humidity spectra have large power at high frequencies. It has been shown that the spectra of temperature and vertical wind have peaks above the Nyquist frequency used here. If the relative humidity spectra are similar, as can be expected, aliasing will be a problem. Therefore, its possible effect on the spectra



must be kept in mind.

A mean curve of the normalized spectra is obtained by using a modified running mean. Graphs of this mean and the actual estimates for the nine short trials are contained in Appendix E. The confidence limits are not drawn because they are too large. The factors required to obtain them are listed with the graphs.

The three unstable trials show indications of two general maxima. The first is at low frequency and is caused by thermal convection. The second is not completely shown in the graphs because its true position is above the Nyquist frequency. It is attributed to mechanical turbulence. The two peaks are in agreement with Webb<sup>1</sup> and Lumley and Panofsky<sup>2</sup>. Trial #8 shows the low frequency peak at the first estimate and, therefore, may be unrepresentative. The peak is small for #1. However, #11 has a well defined peak at slightly higher frequencies. The power in the convective peak of #11 is much larger than the other two. The light winds allow convection to develop to a greater extent and, consequently, more power appears at this frequency. Trial #1 has most of its power at high frequencies. It is felt that the large vertical gradient and the strong wind cause the smaller eddies to have large fluctuations. Therefore, the high frequency fluctuations dominate and result in more power at the mechanical peak.

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<sup>1</sup>E.K. Webb, Autocorrelations and Energy Spectra of Atmospheric Turbulence, C.S.I.R.O., Div. Met. Phys. Tech. Pap. No. 5, Melbourne, Australia, 1955, p. 8.

<sup>2</sup>J.L. Lumley and H.A. Panofsky, The Structure of Atmospheric Turbulence, John Wiley & Sons, New York, 1964, p. 136.



Individual sharp peaks occur throughout each trial. Equation (5.18) and the confidence limits give no significance to these peaks. However, some peaks persist from one trial to another but the magnitude changes. Any large peaks that do persist may be significant but they are more likely happening by chance and by the choice of the sample length. For these reasons, the individual peaks for all the trials are neglected.

A general conclusion for unstable conditions is that the mechanical turbulence peak occurs at a non-dimensional frequency greater than 0.1 and the thermal convection peak occurs below 0.02. The convective peak decreases in frequency and magnitude with increasing wind speed. A large relative humidity gradient with sufficient wind speed causes a magnification of the mechanical peak.

The mean spectra of the three neutral trials still have large power at low frequencies. This is a holdover of the convective power. In this case the power is caused only by changes in the specific humidity. This is accomplished by horizontal inhomogeneity or more likely by large eddies which initiate vertical motion. Once started, air parcels continue in that direction for some time because the restoring buoyancy force is extremely small. This low frequency peak is usually below 0.01. The high frequency ends of the spectra are fairly flat with an indication of a small decline with increasing frequency. The mechanical turbulence peak is extremely flat. The power is still large enough to make aliasing serious.

Again, the confidence limits about the mean spectra yield no significant individual peaks. Repetition occurs by chance without any one frequency being dominant. As before, the individual peaks are



neglected.

The general shape of the neutral spectra is a maximum at low frequency, less than 0.01, with a flat or slowly declining tail at the high frequencies. The low frequency peak is attributed to large, widely spaced, eddies and the mechanical peak is barely apparent.

The most obvious characteristic of the stable spectra is the lack of power at low frequencies. These frequencies are damped by the stable air and have little power. The spectra of #7 and #10 have a sharp rise in the vicinity of 0.1 with #13 a little more gradual. The high frequency power is caused by mechanical turbulence. Its proximity to the Nyquist frequency suggests a severe distortion of the spectra by aliasing.

As in previous cases, the individual peaks are not significant nor persistent. Accordingly, they are neglected.

The conclusions drawn from the stable spectra are the lack of power at low frequency and a mechanical peak at high frequency. There is an absence of convection and large eddies. The mechanical turbulence peak occurs at frequencies beyond 0.6 with a possibility of the peak near a frequency of 0.2 being obscured by aliasing.

Several variables affect the mean spectra. The ones examined here are stability, relative humidity gradient, velocity, and sample length. The effects are determined from the mean spectra of the short and long trials. Heuristic arguments are given to justify the observations.

Stability affects the convective or buoyant portion of the spectra. This peak exists for the unstable and neutral trials and is not evident for the stable trials. This is expected because stable air has no



convection and dampens all the large vertical motions. The low frequency power of the neutral cases is caused by vertical motion which is initiated by large eddies and continues in the absence of a restoring buoyant force. No shift in the high frequency peak can be determined because it is either beyond the upper limit or obscured by aliasing.

The relative humidity gradient has little effect except in trial #1. A large variance is caused by the large gradient and a strong wind. A large proportion of the spectral power is at high frequencies because the turbulent fluctuations are large in magnitude and overpower changes caused by convection or inhomogeneity. This characteristic is expected to repeat itself when the gradient is large and the wind strong.

Major velocity effects are eliminated by using the normalized frequency. One effect which is noticeable is the shift of the convective peak to lower frequencies as the wind increases. The area beneath the peak also decreases. This decrease in power at the low frequencies is due to the inhibiting nature of strong winds on the development of convection. Strong winds also increase the time interval between successive buoyant parcels and cause the peak to shift to lower frequencies.

The sample length effects are quite marked. The change in some of the spectra from short to long trials is of some concern because the high frequency power is drastically reduced. To determine the cause, a close examination of three short trials taken from each of the four long trials was made. The general shapes are approximately the same for any one set of three but individual peaks are random, both in amplitude and frequency. This suggests stability of the mean spectra but not for the detailed spectra. A reduction in the maximum lag of the long trials only



smooths the spectra with no change in the general shape. Because these two factors don't cause any significant changes in the spectra of the long trials, it is felt that these are not responsible for the spectral change from short to long trials. The remaining factor is the different and extra smoothing the long trial raw data received before calculation of the relative humidity data. This smoothing reduces the magnitude of the high frequency fluctuations but has little effect on the low frequencies. The smoothing is the greatest in #7L and #10L and these are the spectra with the most reduction of high frequency power<sup>3</sup>. Trial #1L has little extra smoothing and, consequently, no discernable change in the spectral shape. The examination of the long trials leads to the conclusion that increasing the sample length tends to smooth the spectra but leaves the general shape unchanged. Therefore, the mean spectra of the short trials should be representative of the condition both prior to and after the data sampling.

The minus five-thirds law of the inertial subrange is expected to hold for scalars. Table VIII gives the power to which frequency must be raised to be directly proportional to the spectra. This power is identical to the slope of the straight line when the logarithms of the spectral estimates and frequency are used. Two different methods were used to obtain the slopes of the spectra. Both methods average out to the same value of -0.87. However, the power law applies best to neutral air and for these trials the average slope is -1.13. This is still too small

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<sup>3</sup>The relative humidity spectra of #7L is contained in Appendix E to show the effect of extra smoothing.



Table VIII - Power (Slope) of the Spectral Power Law

No. <sup>†</sup>	Stab.	$b_1^*$	$b_2^*$
1	u	-0.65	-0.58
8	u	-0.87	-0.83
11	u	-1.00	-0.95
3	n	-1.16	-1.20
6	n	-0.90	-0.97
9	n	-1.30	-1.26
7	s	-0.81	-0.77
10	s	-0.38	-0.48
13	s	-0.73	-0.78

<sup>†</sup> The long trials are not included because their spectra are in error.

\*Slope  $b_1$  is calculated by a weighted two point method and  $b_2$  is calculated using the least squares method.

to obey the minus five-thirds law. Two possible approaches are available. First, is this the correct frequency range? In view of the theory and previous experiments it is reasonable to reply affirmatively. Secondly, the problem of aliasing is present. This is the most reasonable explanation when it is realized that aliasing is large and is most serious at the highest frequency. This could cause the observed decrease in the magnitude of the slope. Also, the temperature spectra do not agree with the minus five-thirds law. Therefore, it is concluded that the minus five-thirds law for the relative humidity spectra is not disproved.

The spectral characteristics consist of a bimodal shape except for stable conditions where the convective peak is nonexistent. The mechanical peak is generally at frequencies larger than the Nyquist frequency. The individual peaks are random, are not significant, and



are smoothed out with increasing sample length. The shape of the mean spectra is affected by the stability, occasionally by the relative humidity gradient, but not by the sample length. The wind speed affects the size and frequency of the convective peak. Aliasing occurs and is serious enough to make the analysis of the minus five-thirds law less meaningful.

#### Spectral Relations

The relative humidity spectra are compared with the temperature spectra for all nine short trials. At very low frequencies, such as one cycle per day, temperature is the controlling factor of the relative humidity. This relation is not true at high frequencies so a cut-off must exist. A visual comparison of the spectra was done to determine the frequency at which the above relation no longer holds<sup>4</sup>. Equation (5.20) is also used to establish this cut-off frequency, or the frequency range in which the spectra are similar. The values of D and the frequency ranges are listed in Table IX. The five per cent level of significance for D is 0.0628. A visual comparison of the relative and specific humidity spectra eliminated five of the nine trials from the calculation of D. The remaining four are included in Table IX<sup>5</sup>. No similarities were found between the relative humidity and vertical wind spectra. In eleven of the thirteen cases listed, the spectra are similar from the lowest frequency in the analysis to some cut-off frequency. The other two cases, which are neutral, showed no similarity as is expected. For

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<sup>4</sup>The temperature spectrum for #10 is included in Appendix E.

<sup>5</sup>The specific humidity spectrum for #9 is included in Appendix E.



Table IX - Frequency Range for Relative Humidity and Both Temperature and Specific Humidity.

No.	Stab.	$S_2'$	Wind	$f_{c1}^*$	$f_{c2}^*$	D
1	u	T	7.99	0.003-0.05	0.003-0.07	0.0548
8	u	T	9.31	0.002-0.07	0.002-0.05	0.0480
11	u	T	3.80	0.005-0.14	0.005-0.16	0.0570
3	n	T	4.29	None	None	--
6	n	T	8.15	None	None	--
9	n	T	6.20	0.003-0.07	0.003-0.09	-0.0597
7	s	T	7.12	0.003-0.10	0.003-0.10	0.0601
10	s	T	1.65	0.01-0.59	0.01-0.56	0.0624
13**	s	T	1.48	0.01-0.66	0.01-0.65	0.0575
3**	n	q	4.29	0.005-0.23	0.005-0.23	0.0506
6	n	q	8.15	0.003-0.12	0.003-0.12	-0.0435
9	n	q	6.20	0.002-0.16	0.002-0.16	-0.0194
10	s	q	1.65	0.01-0.59	0.01-0.59	-0.0346

'  $S_2$  is the second spectra used for the comparison.

\* $f_{c1}$  and  $f_{c2}$  are the visual and calculated frequency ranges.

\*\* One of the spectra is multiplied by a constant to correct its area to unity.

temperature and relative humidity, the cut-off frequency seems to be affected by two factors: the stability and the wind. The wind speed is the more dominant as shown by the large cut-offs for light wind trials, #10 and #13. This is also shown by comparing #1 and #8 with #11. The stability effect is much smaller and may be just coincidence. However, trials #1, #9, and #7 have similar winds and they show a slight increase in the cut-off frequency as stability increases. Comparing the relative and specific humidity spectra, the same wind effect is apparent in #6, #9, and #3. No stability effect is determinable due to the lack of trials with similar wind speeds and spectra.

The conclusions drawn from the above data are: except for neutral



conditions, the temperature controls the relative humidity up to a certain frequency; for neutral and some stable conditions the specific humidity controls the relative humidity up to a given frequency; this cut-off frequency depends on the wind speed and the stability.

### Conclusions

Relative humidity fluctuations can be described by their amplitude and frequency. The standard deviation, a measure of amplitude, can be determined from steady state variables. The sign of the skewness is obtained from the stability so that its effect on the fluctuations can be taken into account. The major frequencies of the fluctuations, as shown by the spectra characteristics, can be estimated from the stability and the wind speed. Lastly, the frequency range, in which the relative humidity is controlled by other factors, can also be determined from the stability and wind speed. Therefore, it should be possible to specify the amplitude and the frequency of relative humidity fluctuations from steady state parameters.



## CHAPTER VII

### FLUX ANALYSIS

The vertical fluxes of heat and moisture were calculated from Equations (4.4) and (4.5). This was accomplished by a computer program and the results are listed in Table X. The coefficients of eddy transfer, obtained from Equations (4.6) and (4.7), are also given in Table X. The ratio of the heat and moisture coefficients is included. The data required to solve the equations are listed in Table XI. The specific humidity gradient, like the relative humidity gradient, is an estimate. All available data were used to obtain this estimate which, combined with the temperature gradient, yields the relative humidity gradient used previously.

Several errors are involved in the value of the coefficients. Small fluxes and gradients are susceptible to large percentage errors. This is caused by errors in the data being of the same magnitude as the calculated fluxes. The coefficient of diffusivity for all nine trials is based on the estimates of the specific humidity gradient. The magnitude of the error is unknown and no correction can be made. This error is best illustrated by the sign and the magnitude of the moisture coefficients for trials #7 and #13. Combined, these two errors cause suspicious eddy transfer coefficients which must be disregarded. Such coefficients are not underlined in Table X. The average heat and moisture coefficients are obtained from the underlined values. The respective averages are 1970 and 2110. Although the ratios for individual trials vary from 0.06 to 10.10, the above averages are approximately equal with a ratio between them of 0.93. However, the small sample and the possible errors make it difficult to draw a reliable conclusion



Table X - Eddy Fluxes and Coefficients

No.	Stab.	$F_h$ (cal/cm <sup>2</sup> sec)	$F_w$ (g/cm <sup>2</sup> sec)	$K_h$ (cm <sup>2</sup> /sec)	$K_w$ (cm <sup>2</sup> /sec)	$K_h/K_w$
1	u	$44.2 \times 10^{-4}$	$2.64 \times 10^{-6}$	<u><math>2.63 \times 10^3</math></u>	<u><math>1.07 \times 10^3</math></u>	2.45
8	u	25.3	0.622	<u>2.07</u>	<u>0.52</u>	3.98
11	u	22.6	0.688	<u>1.36</u>	<u>3.88</u>	0.35
3	n	0.260*	2.34	<u>0.71</u>	<u>2.07</u>	0.34
6	n	-1.08	-0.957	4.46	<u>3.46</u>	1.29
9	n	-0.717*	1.12	16.96	<u>1.68</u>	10.10
7	s	-6.60	0.557	<u>1.83</u>	-7.64	-0.24
10	s	-0.516*	0.0574*	0.05	0.78	0.06
13	s	-0.0433*	0.0308*	-0.005	-12.14	--

\* Accuracy is low because the values are small.

— Values that have the best reliability.

Table XI - Data for Calculation of Coefficients

No.	T (°K)	p (mb)	d (g/cm <sup>3</sup> )	r <sub>T</sub> (°C/m)	r <sub>q</sub> (g/kgm)
1	291.1	934.1	$1.12 \times 10^{-3}$	0.63	-0.22
8	297.0	928.3	1.09	0.48	-0.11
11	302.0	939.8	1.08	0.65	-0.016
3	300.0	922.4	1.07	0.024*	-0.11
6	305.4	929.3	1.06	0.0 *	0.026
9	292.6	929.6	1.11	0.008*	0.060
7	294.0	929.2	1.10	-0.13	0.0066*
10	285.4	930.8	1.14	-0.35	0.0064*
13	295.2	937.6	1.11	-0.32	0.00023*

\* Accuracy is low because the values are small.



about the eddy transfer mechanisms of heat and moisture.

The underlined coefficients and the fluxes in Table X are similar to those obtained by Swinbank<sup>1</sup>. Priestley<sup>2</sup> quotes  $K_h$  between 2000 and 2400 at a height of two metres. The average values agree well with these.

Sutton<sup>3</sup> states that the heat coefficient is not constant but increases with height. Only one height is involved here so a constant coefficient should be obtained. The average values given are 500 to 1300 for one to seven metres. These are smaller than the averages stated above. The ratio of the heat and moisture coefficients is shown to be near or equal to unity by Munn<sup>4</sup>, Dyer<sup>5</sup>, Swinbank and Dyer<sup>6</sup>, and Crawford<sup>7</sup>. Considering the theoretical work, previous experimental results, and the possible error, the coefficients and their ratio obtained from these trials are reasonable. However, quantity and accuracy are not sufficient to make

<sup>1</sup>W.C. Swinbank, An Experimental Study of the Eddy Transports in the Lower Atmosphere, C.S.I.R.O., Div. Met. Phys. Tech. Pap. No. 2, Melbourne, Australia, 1955, pp. 11-14.

<sup>2</sup>C.H.B. Priestley, Turbulent Transfer in the Lower Atmosphere, University of Chicago, Chicago, 1959, p. 104 and p. 108.

<sup>3</sup>O.G. Sutton, Micrometeorology, McGraw-Hill, New York, 1953, p. 214.

<sup>4</sup>R.E. Munn, Descriptive Micrometeorology, Academic Press, New York, Advances in Geophysics, Supplement 1, 1966, pp. 94-95.

<sup>5</sup>A.J. Dyer, "The Turbulent Transport of Heat and Water Vapour in an Unstable Atmosphere", Quart. Journ. of Roy. Met. Soc., Vol. 93, 1967, p. 507.

<sup>6</sup>W.C. Swinbank and A.J. Dyer, "An Experimental Study in Micrometeorology", Quart. Journ. of Roy. Met. Soc., Vol. 93, 1967, pp. 499-500.

<sup>7</sup>T.V. Crawford, "Moisture Transfer in Free and Forced Convection", Quart. Journ. of Roy. Met. Soc., Vol. 91, 1965, p. 26.



a definite conclusion.

Insufficient number of trials and the inaccuracy of the coefficients obscure any effects that stability, wind speed, wind shear, standard deviation, or skewness may have on the coefficients or their ratio. Therefore, no attempt can be made to determine any of the relationships.

Several errors are included in the calculation of the eddy fluxes. One error that can be estimated is the one caused by the slow response of the sensors. High frequencies fluctuations are not measured and, therefore, their contribution to the vertical transfer is not included in the flux. Deacon<sup>8</sup> showed the error involved could be large and depends on the stability, the averaging time of the data, the wind speed, and the height of observation. The error increases with averaging time, wind speed, and stability, but decreases with increasing height. To measure eighty per cent of the flux, the quantity  $tV/Z$  must be less than one. For an averaging time,  $t$ , of one second and a mean wind of eight m. per sec. the observation height must be eight metres. With this consideration, the fluxes in Table X have large errors, and should be corrected using the graph given by Deacon<sup>9</sup>. Future flux measurements must be designed to reduce this error caused by the slow instrument response.

The error involved in the estimation of the specific humidity

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<sup>8</sup>E.L. Deacon, "The Measurement of Turbulent Transfer in the Lower Atmosphere", Advances in Geophysics, Vol. 6, 1959, p. 213.

<sup>9</sup>Loc. cit.



gradient is unknown but it has a large effect on the value of the coefficient. However for the larger gradients, the error is estimated to be less than 50 per cent, with a corresponding 50 per cent error in the eddy coefficients. Considering the magnitude of this error, the underlined coefficients in Table X have realistic values.

The instruments and methods used here to measure the fluxes and to calculate the heat and moisture coefficients of eddy transfer could be modified and improved to yield more reliable results.



## CHAPTER VIII

### CONCLUSIONS

The fluctuations of relative humidity, important to certain scientific interests, are described in terms of statistical and spectral properties. The vertical fluxes of heat and moisture, along with the eddy coefficients of conductivity and diffusivity, are involved in the heat balance of the atmosphere and in transfer mechanisms of turbulence. An attempt to extract the major results and important relations of the previous chapters will be made in this chapter.

#### Relative Humidity Spectral Analysis

Three Equations, (5.1), (5.8), and (5.13), were derived to express the relative humidity standard deviation in terms of steady state parameters. The second equation is the most practical because it does not rely on the knowledge of the vertical wind standard deviation. The last equation is useful only in diagnosis because it requires three quantities which are extremely difficult to estimate without actual measurements. Therefore, the standard deviation is best specified by Equation (5.8) which requires a constant, a time variable, the wind speed, and the vertical gradient.

The relative humidity skewness study yielded little of practical importance except for the relation between skewness sign and stability and the effect of skewness on the amplitude of fluctuations.

It was discovered that the relative humidity spectra do not obey the minus five-thirds power law. Because this seems to contradict the theory, the error is attributed to the effect of aliasing at the higher frequencies. The comparison of temperature and specific humidity with



relative humidity spectra is striking. Under certain conditions and in specific frequency bands, the two spectra are almost identical. This suggests that one factor is dominant in controlling the relative humidity.

The division of the normalized variance among the non-dimensional frequency bands resulted in an average curve for each stability case. The effect of wind speed and relative humidity vertical gradient on the convective and mechanical peaks is noted. All the sharp individual peaks in the spectra are attributed to the specific sample. They are not significant according to a confidence limit test and are shown to decrease in importance with increased sample length. It is also stated that aliasing is a serious problem and is most likely to have a profound effect on the spectral estimates.

#### Heat and Moisture Fluxes

The calculated values of the fluxes are reliable only when there are sufficient vertical gradients and wind. Otherwise, the covariance of the vertical wind and temperature or specific humidity is small in magnitude and, consequently, susceptible to a large percentage error. To achieve good estimates for the eddy coefficients of conductivity and diffusivity accurate vertical gradients are required. The failure to do this, in respect to the specific humidity, leaves the diffusivity coefficient values open to large errors. Furthermore, the small sample size, used to obtain ratios of the coefficients, reduces the justification for any conclusion. For these reasons, no statement can be made concerning the transfer mechanisms of heat and water vapour.

#### Suggestions for Future Research

The instruments and methods used for the collection of data and



its analysis are theoretically sound. Practical application to the atmosphere is the source of many problems. Some of these were eliminated during the initial trials but many remain to be solved.

The thermocouples were accurately calibrated and their characteristics were sufficiently determined. However, analysis of the wind speed data revealed that certain errors in the magnitude of the speed were involved. These resulted from an electrical output for the wind speed when it should have been zero and from an error in the reference level when the electrical output was recorded. The wind speed data could be improved by a more accurate calibration of the vectorvane and by taking more care in determining the recording reference level.

A serious drawback of the measured mean properties was the inaccurate specific and relative humidity gradients. More precise instrumentation is required to measure these gradients.

Another desirable improvement to the sensors is a smaller time constant for the fast-response sensors. This would allow a shorter averaging time for the digitization which, correspondingly, extends the spectra to higher frequencies. Increasing the observation height also extends the spectra to higher non-dimensional frequencies. The result of these changes may take the spectra beyond the mechanical turbulence peak and, thereby, reduce the aliasing problem. Also, these changes would improve the accuracy of the calculated fluxes.

More stable spectral estimates are attainable by using longer sample times. Even the 2000 second trials still retained a degree of randomness about the running mean average. Therefore, a sample time of one hour may be more realistic in obtaining a stable smooth spectrum.



A method to decrease significantly the digitizing and computing time was being devised at Suffield. This involved a direct reading of all the channels on the magnetic tape into the computer. Also, this assures a simultaneous start for all the time series. More sophisticated computer programs could be written to do all the calculations at one time. This procedure would make the output available earlier and would allow a quicker analysis of the spectra to be done.

#### Summary

With the above suggestions and other improvements, the instrumentation and methods described in this thesis could be made suitable to yield reliable spectral estimates and accurate measurements of the vertical fluxes. The more dependable data could be used to improve further the relations developed for the specification of relative humidity fluctuations. Also, the data would be of greater value in the study of eddy transfer of heat and moisture.



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## APPENDIX A

## MEAN DATA OF THE TRIALS

RH TRIAL NO. 1DATE June 21, 1967ZERO 1055 LSTLOCATION 300' towerSYNOPTIC SITUATION 1/10 thin AC, 2/10 thin CI, wind NE, unstable.PRESSURE 934.1 mb

REMARKS No feed back on difference so voltage was reduced by half to get it on the magnetic tape. Light rain on June 16 and 17 amounted to 0.62 in. Light shower at DRES at 0915 LST on the 21st with 0.01 in. AC shadow at 35 min. mark of record. Both temperature and diff. slopes positive. Wind blowing through tower to anemometers.

<u>TIME (min)</u>	<u>PSYCHROMETER °F</u>			<u>GRADIENT in °F (4-1/2m)</u>			<u>WIND (mph)</u>			
	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4</u>	-	<u>Tw1/2</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>
-5	66.0	56.2	9.8	--	missing			--	--	--
0	66.1	55.5	10.6	-4.1	"			17.6	17.9	18.6
5	64.5	54.0	10.5	--	"			--	--	--
10	64.2	54.0	10.2	--	"			--	--	--
15	64.7	54.0	10.7	-3.9	"			15.7	15.4	16.2
20	65.0	54.3	10.7	--	"			--	--	--
25	65.2	54.5	10.7	--	"			--	--	--
30	65.7	54.6	11.1	-4.2	"			15.3	15.8	15.6
35	64.6	54.0	10.6	--	"			--	--	--
40	65.3	54.8	10.5	--	"			--	--	--
45	64.7	53.8	10.9	-3.9	"			15.0	15.2	15.0
50	64.9	53.7	11.2	--	"			--	--	--
55	65.9	54.3	11.6	--	"			--	--	--
60	65.9	54.2	11.5	-4.9	"			14.8	14.5	15.4

Estimated 4m - 0.5m gradients for 0 to 15 min.

$$\begin{aligned}
 Rh &= 1.72 \\
 q &= -0.770 \\
 e &= -1.16
 \end{aligned}$$



RH TRIAL NO. 3DATE July 6, 1967ZERO 1735 LSTLOCATION 300' towerSUNSET 2035 LSTSYNOPTIC SITUATION 1/10 thin CU, 1/10 thin AC, wind SW, neutral.PRESSURE 922.4 mb.

REMARKS Feed back on difference. Showers at DRES on the morning and evening of July 5th amounted to 0.05 and 0.09 in. respectively. During the trial the sun was partially under AC at the 10 min. mark, all under at 29 min., partially out at 40 min., all out at 41 min., partially under at 45 min., and all under the AC at the 48 min. mark. Tw gradient uncorrected and measured by one wet bulb. Both temp. and diff. slopes negative for rest of trials.

	<u>PSYCHROMETER °F</u>			<u>GRADIENT in °F (4-1/2)</u>			<u>WIND (mph)</u>		
<u>TIME(min)</u>	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4 -</u>	<u>Tw1/2</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>
-10	82.1	57.2	24.9	--	--	59.0	--	--	--
-5	83.2	57.4	25.8	-.32	--	60.0	--	--	--
0	83.0	57.5	25.5	-.54	57.8	59.0	--	--	--
5	82.7	57.2	25.5	-.54	58.0	58.8	12.9	17.4	18.7
10	81.5	57.6	23.9	-.54	57.5	58.6	--	--	--
15	80.4	57.3	23.1	-.54	58.0	--	--	--	--
20	81.3	58.1	23.2	-.46	57.9	58.1	9.9	13.4	14.2
25	81.3	57.9	23.4	-.46	58.1	58.0	--	--	--
30	80.8	57.4	23.4	-.27	(58.8)	58.2	--	--	--
35	80.1	57.4	22.7	-.11	(58.7)	58.3	9.4	12.7	13.6
40	80.1	57.5	22.6	-.08	(58.8)	58.4	--	--	--
45	81.0	57.4	23.6	-.24	--	59.1	--	--	--
50	81.0	57.4	23.6	-.16	--	58.4	9.6	12.5	13.5
55	80.2	57.1	23.1	-.22	--	58.1	--	--	--

Values in brackets are estimates

Estimated 4m - 0.5m gradients for 30 to 40 min.

RH = -1.44

q = -0.370

e = -0.552



RH TRIAL NO. 6DATE July 12, 1967ZERO 1801 LSTLOCATION Rocket layoutSUNSET 2031 LSTSYNOPTIC SITUATION 8/10 CI, wind E, neutral.PRESSURE 929.3 mb.

REMARKS 16 °F on difference feed back. Sun under CI except partially out at 15 min. mark for one min. and 25 min. mark for about five min. No precipitation since the 5th. Tw gradient uncorrected.

	<u>PSYCHROMETER °F</u>			<u>GRADIENT in °F (4-1/2m)</u>			<u>WIND (mph)</u>		
<u>TIME (min)</u>	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4</u> (uncorrected)	<u>- Tw1/2</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>
-5	81.3	60.6	20.7	-.29	--	61.9	--	--	--
0	80.9	60.8	20.1	-.27	--	61.8	--	--	--
5	80.8	60.3	20.5	-.13	--	61.5	--	--	--
10	80.4	60.1	20.3	0	--	61.2	17.4	22.2	25.1
15	80.2	60.1	20.1	0	--	61.1	--	--	--
20	80.0	60.0	20.0	0	--	61.0	17.7	21.5	24.0
25	80.1	60.2	19.9	-.27	--	61.3	--	--	--
30	79.9	60.1	19.8	-.13	--	61.0	18.8	23.7	25.5
35	80.0	60.1	19.9	-.18	--	61.2	--	--	--

Estimated gradients 4m - 0.5m for 10 to 25 min. from 2 and 0.5m data.

RH = 0.379

q = 0.0913

e = 0.132



RH TRIAL NO. 7DATE July 12, 1967ZERO 2100 LSTLOCATION Rocket layoutSUNSET 2031 LSTSYNOPTIC SITUATION 3/10 CI, wind E, stable.PRESSURE 929.2 mb

REMARKS 8 °F feed back on difference. Temp. gradient after 30 min. mark doubtful. Radio transmission approximately every 15 min. starting on the hour. No precipitation since the 5th.

	<u>PSYCHROMETER °F</u>			<u>GRADIENT in °F (4-1/2m)</u>			<u>WIND (mph)</u>			
<u>TIME (min)</u>	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4</u>	-	<u>Tw1/2</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>
-10	72.8	57.8	15.0	.8		missing		--	--	--
-5	71.8	57.9	13.9	.8	"			--	--	--
0	71.2	57.2	14.0	.8	"			17.0	21.5	24.4
5	70.9	57.1	13.8	.8	"			--	--	--
10	70.2	57.0	13.2	.8	"			--	--	--
15	70.0	57.0	13.0	.8	"			16.3	21.0	23.6
20	69.9	56.7	13.2	.8	"			--	--	--
25	69.6	56.5	13.1	.8	"			--	--	--
30	69.4	56.3	13.1	.8	"			15.7	20.4	23.2
35	missing									
40	69.3	56.0	13.3	.8	"			--	--	--
45	69.1	56.1	13.0	.9	"			15.2	19.6	22.4
50	68.0	55.6	12.4	.9	"			--	--	--
55	68.1	55.4	12.7	1.0	"			--	--	--
60	67.8	55.1	12.7	1.0	"			15.2	19.3	21.8
65	67.4	55.1	12.3	1.1	"			--	--	--
70	67.5	55.0	12.5	1.0	"			--	--	--
75	67.1	55.0	12.1	1.1	"			--	--	--

Estimated gradients 4m - 0.5m for 20 to 30 min. from trial 6.

$$\begin{aligned} RH &= -1.11 \\ q &= 0.0232 \\ e &= 0.0305 \end{aligned}$$



RH TRIAL NO. 8

DATE August 2, 1967

ZERO 1500 LST

LOCATION Rocket layout

SYNOPTIC SITUATION 3/10 CU, wind NW, unstable.

PRESSURE 928.3 mb

REMARKS 12 °F feed back on difference. Tw gradient uncorrected. Thunderstorms were frequent in the late afternoon and evening of July 31st with only 0.01 in. at DRES but 0.25 to 0.50 estimated over the area. First min. of temp. and diff. are missing and first 10 min. are doubtful, also doubtful at the 22 min. mark. A few Cu shadows at the 25 and 28 min. mark.

<u>TIME(min)</u>	<u>PSYCHROMETER °F</u>			<u>GRADIENT in °F (4-1/2m)</u>			<u>WIND (mph)</u>		
	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4 - Tw1/2 (uncorrected)</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>	
-15	74.8	55.0	19.8	-3.4	--	--	--	--	--
-10	72.8	54.0	18.8	-2.2	56.0	--	--	--	--
-5	73.8	54.3	19.5	-2.7	55.8	56.7	--	--	--
0	74.9	54.5	20.4	-3.2	55.7	--	--	--	--
5	74.3	54.3	20.0	-3.2	55.7	57.2	--	--	--
10	75.3	54.9	20.4	-3.1	55.8	57.4	--	--	--
15	75.0	54.8	20.7	-3.0	56.5	57.9	19.1	24.3	25.9
20	74.8	54.1	20.7	-2.4	55.2	57.2	--	--	--
25	74.0	54.1	19.9	-2.3	55.6	57.0	--	--	--
30	75.3	54.8	20.5	-2.7	55.3	57.2	20.4	25.8	26.8
35	74.4	53.1	21.3	-3.0	54.2	56.2	--	--	--

Gradients 4m - 0.5m for 10 to 15 min.

$$RH = 0.714$$

$$q = 0.383$$

$$e = -0.575$$



RH TRIAL NO. 9DATE August 2, 1967ZERO 1850 LSTLOCATION Rocket layoutSUNSET 2006 LSTSYNOPTIC SITUATION Clear, wind NNW, neutral.PRESSURE 929.6 mb

REMARKS 10 °F feed back on difference. Tw gradient uncorrected. Thunderstorms were frequent in the late afternoon and evening of July 31st with only 0.01 in. at DRES but 0.25 to 0.50 in. estimated over the area. Radio transmission at 14 to 15 min. mark.

	<u>PSYCHROMETER °F</u>			<u>GRADIENT in °F (4-1/2)</u>			<u>WIND (mph)</u>		
<u>TIME (min)</u>	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4</u>	<u>- Tw1/2</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>
-15	68.5	51.0	17.5	-.45	53.0	52.6	--	--	--
-10	68.1	51.1	17.0	-.40	52.5	53.0	--	--	--
-5	68.0	51.1	16.9	-.32	52.9	52.8	--	--	--
0	67.8	50.0	17.8	-.21	50.9	51.9	--	--	--
5	67.1	49.6	17.5	-.10	50.4	50.9	--	--	--
10	67.0	49.8	17.2	-.08	51.0	51.0	13.9	18.3	18.7
15	66.9	50.0	16.9	.03	50.9	51.1	--	--	--
20	66.3	49.8	16.5	.09	51.0	51.0	14.9	17.1	17.3
25	66.0	50.0	16.0	.17	51.1	50.7	--	--	--
30	65.8	50.0	15.8	.19	51.3	50.8	9.6	15.4	15.9
35	65.2	49.1	15.5	.28	51.0	50.4	--	--	--

Gradients 4m - 0.5m for 5 to 15 min.

$$\begin{aligned}
 RH &= -1.40 \\
 q &= -0.210 \\
 e &= -0.316
 \end{aligned}$$



RH TRIAL NO. 10DATE August 2, 1967ZERO 2045 LSTLOCATION Rocket layoutSYNOPTIC SITUATION Clear, wind N, stable.SUNSET 2006 LSTPRESSURE 930.8 mb

REMARKS 4 °F feed back on difference. Tw gradient uncorrected. Thunderstorms were frequent in the late afternoon and evening of July 31st with only 0.01 in. at DRES but 0.25 to 0.50 in. estimated over the area. Radio transmissions at 15 min. intervals but from a removed point.

	PSYCHROMETER °F			GRADIENT in °F (4-1/2m)			WIND (mph)			
<u>TIME (min)</u>	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4</u> (uncorrected)	<u>-</u>	<u>Tw1/2</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>
-15	57.4	44.4	13.0	1.2	46.0	44.2	--	--	--	--
-10	57.0	44.0	13.0	1.2	46.2	44.0	--	--	--	--
-5	56.0	43.8	12.2	1.3	44.3	43.4	--	--	--	--
0	54.8	43.4	11.4	1.4	--	--	5.7	--	13.1	
5	54.0	43.2	10.8	2.2	44.3	43.1	--	--	--	
10	54.2	44.1	10.1	2.2	45.0	53.9	--	--	--	
15	54.1	44.0	10.1	2.2	--	--	3.7	--	11.2	
20	missing									
25	missing									
30	53.7	44.2	9.5	3.3	--	--	3.6	9.1	11.8	
35	54.0	45.2	8.8	3.3	47.0	45.0	--	--	--	
40	54.1	46.1	8.0	2.8	47.2	45.9	--	--	--	
45	53.8	46.2	7.6	2.3	47.0	45.9	4.2	9.6	11.8	
50	52.4	45.7	6.7	2.1	46.7	45.4	--	--	--	
55	52.7	46.0	6.7	2.2	46.7	45.7	--	--	--	
60	52.9	46.0	6.9	2.6	47.0	45.3	2.9	8.7	9.4	
65	51.9	45.9	6.0	3.2	47.0	45.0	--	--	--	

Gradients 4m - 0.5m for 5 to 15 min

RH = -3.78

q = -0.0225

e = -0.0364



RH TRIAL NO. 11DATE August 30, 1967ZERO 1500 LSTLOCATION 300' towerSYNOPTIC SITUATION 1/10 thin CU, wind SE, unstable.PRESSURE 939.8 mbREMARKS 16 °F feed back on diff. Tw gradient uncorrected. No rain since August 7.

	<u>PSYCHROMETER °F</u>			<u>GRADIENT in °F (4-1/2)</u>			<u>WIND (mph)</u>			
<u>TIME (min)</u>	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4</u>	<u>-</u>	<u>Tw1/2</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>
-5	83.6	58.9	24.7	-4.3	61.0		60.9	--	--	--
0	84.0	58.8	25.2	-4.3	60.5		61.2	--	--	--
5	84.3	58.8	25.5	-4.4	60.2		61.7	--	--	--
10	84.3	58.9	25.4	-4.3	60.4		61.1	8.8	11.3	12.1
15	83.5	58.3	25.2	-4.1	59.9		61.6	--	--	--
20	83.0	58.2	24.8	-3.9	60.9		61.2	8.5	10.3	10.9
25	84.0	58.8	25.2	-4.0	60.9		61.1	--	--	--
30	84.2	58.9	25.3	-4.1	60.2		61.1	9.4	11.8	11.9

Gradients 4m - 0.5m for 10 to 20 min.

$$\begin{aligned} RH &= 2.46 \\ q &= -0.0575 \\ e &= -0.0900 \end{aligned}$$



RH TRIAL NO. 13DATE August 30, 1967ZERO 1855 LSTLOCATION 300' towerSUNSET 1912 LSTSYNOPTIC SITUATION Clear, wind SE, stablePRESSURE 937.6 mb.REMARKS 14 °F feed back on diff. Tw uncorrected. No rain since August 7.

	<u>PSYCHROMETER °F</u>			<u>GRADIENT in °F (4-1/2m)</u>			<u>WIND (mph)</u>			
<u>TIME (min)</u>	<u>T</u>	<u>Tw</u>	<u>Diff</u>	<u>Temp</u>	<u>Tw4</u> (uncorrected)	<u>-</u>	<u>Tw1/2</u>	<u>2m</u>	<u>8m</u>	<u>16m</u>
0	73.0	54.8	18.2	+1.5	55.7		54.1	--	--	--
5	72.1	54.5	17.6	+2.2	55.7		54.0	--	--	--
10	70.2	53.8	16.4	+2.4	54.3		52.6	3.3	6.5	9.8
15	70.0	53.1	16.9	+3.5	54.1		52.2	--	--	--
20	69.0	53.2	15.8	+3.5	54.0		51.9	3.4	6.3	9.7

Gradients 4m - 0.5m for 0 to 10 min.

$$\begin{aligned} Rh &= -2.22 \\ q &= +.0008 \\ e &= -.0090 \end{aligned}$$

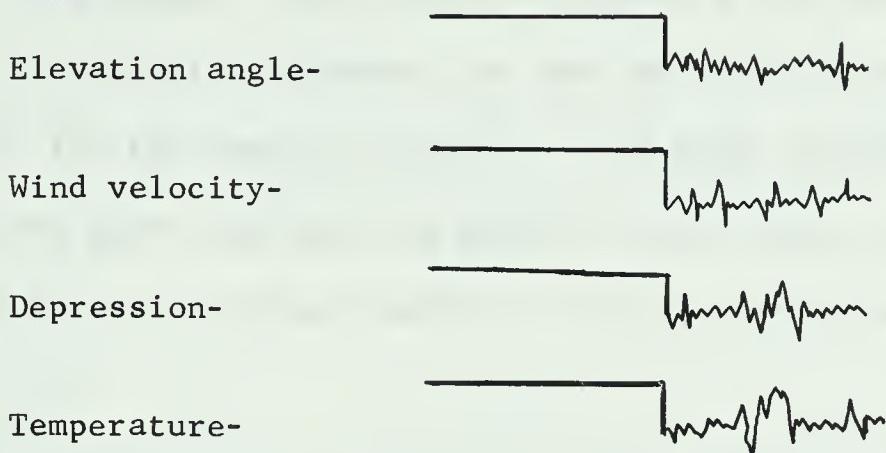


## APPENDIX B

Example of Determination of Start of Digitized Data on Paper Punch Tape.

RH Trial No. 1

Plot of Data.



Data Units. (eight counts per sec.)

Critical  
No.

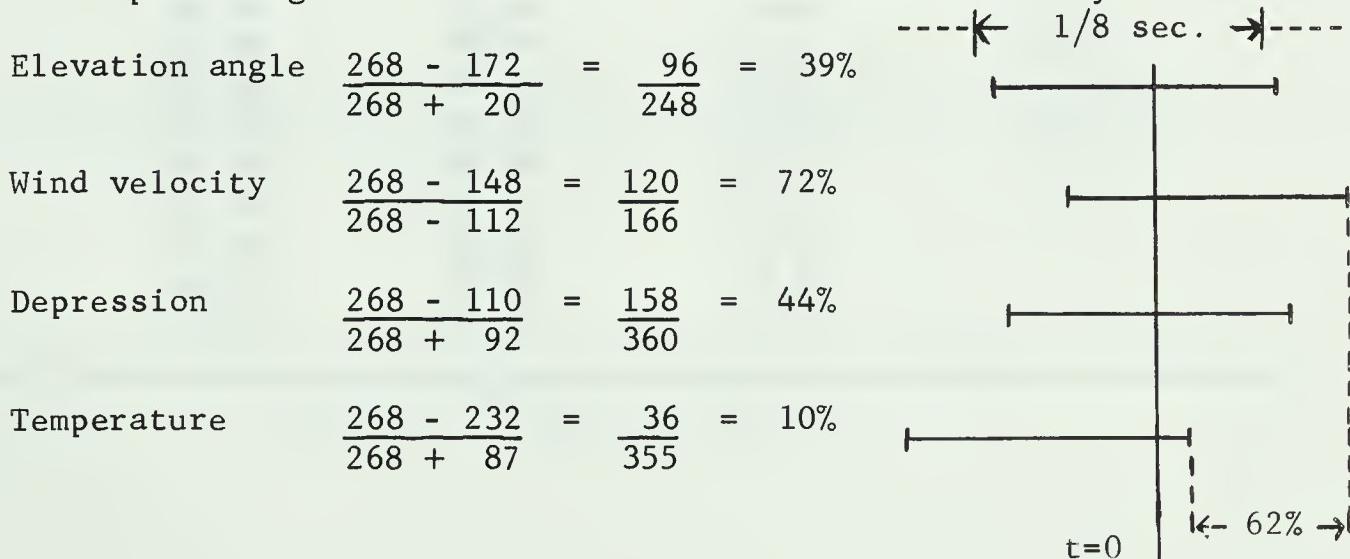
Elevation angle 268, 268, 268, 268, 268, 172', 20, 20, 4, -3, 20

Wind velocity 268, 268, 268, 268, 268, 148', 112, 112, 112, 112, 112

Depression 268, 268, 268, 268, 268, 110', -92, -92, -92, -108, -124

Temperature 268, 268, 268, 268, 268, 232', -87, -107, -115, -95, -75

Approximate percentage of trial in critical no. and time delay



The difference in start times is  $(.62 \times 1/8)$  approximately 0.08 sec.

Similar or better results were obtained for the other trials.



## APPENDIX C

## GRAPHS OF THE DATA

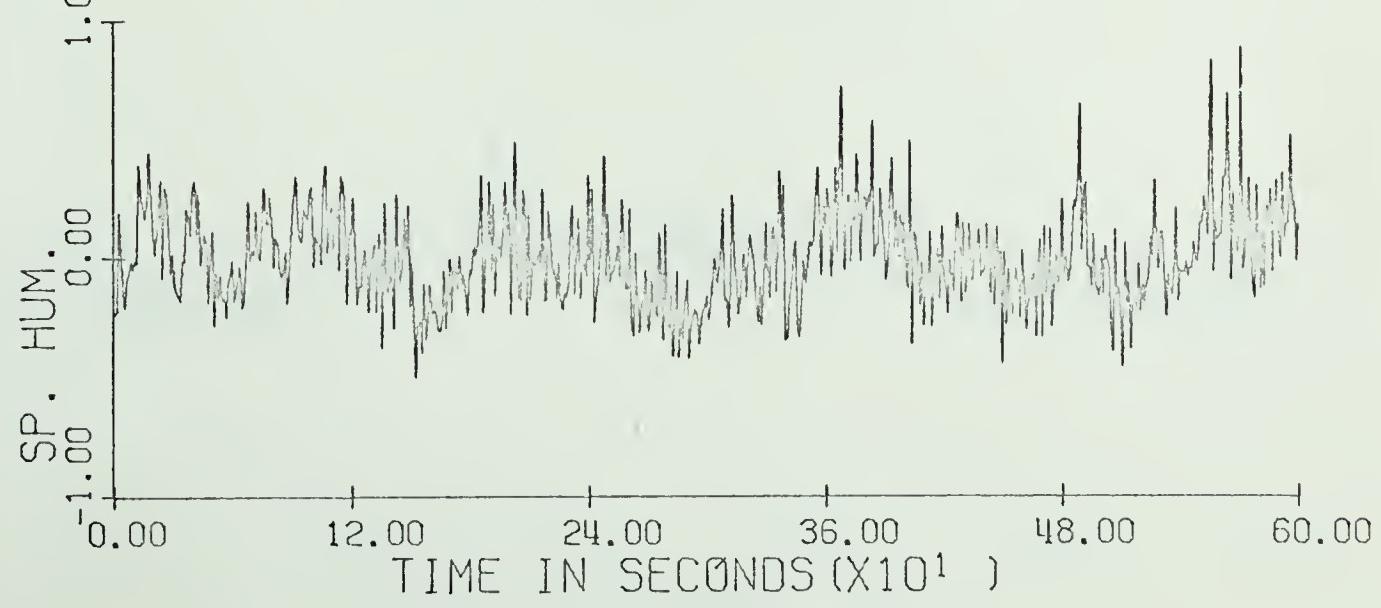
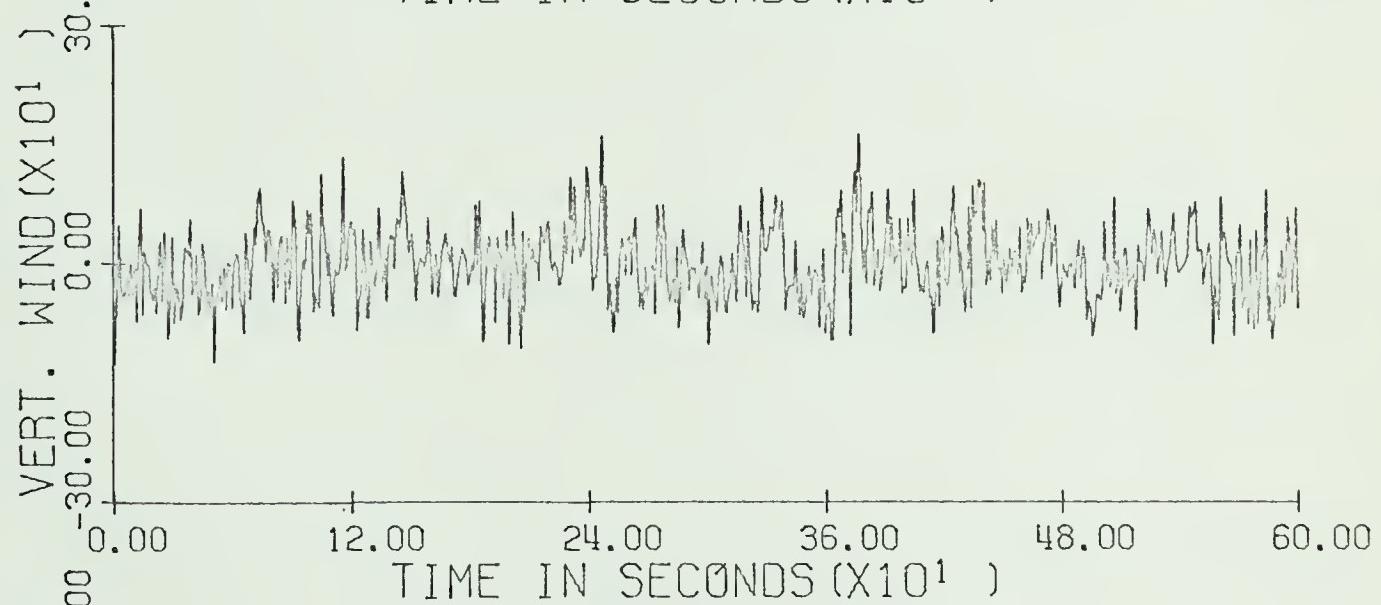
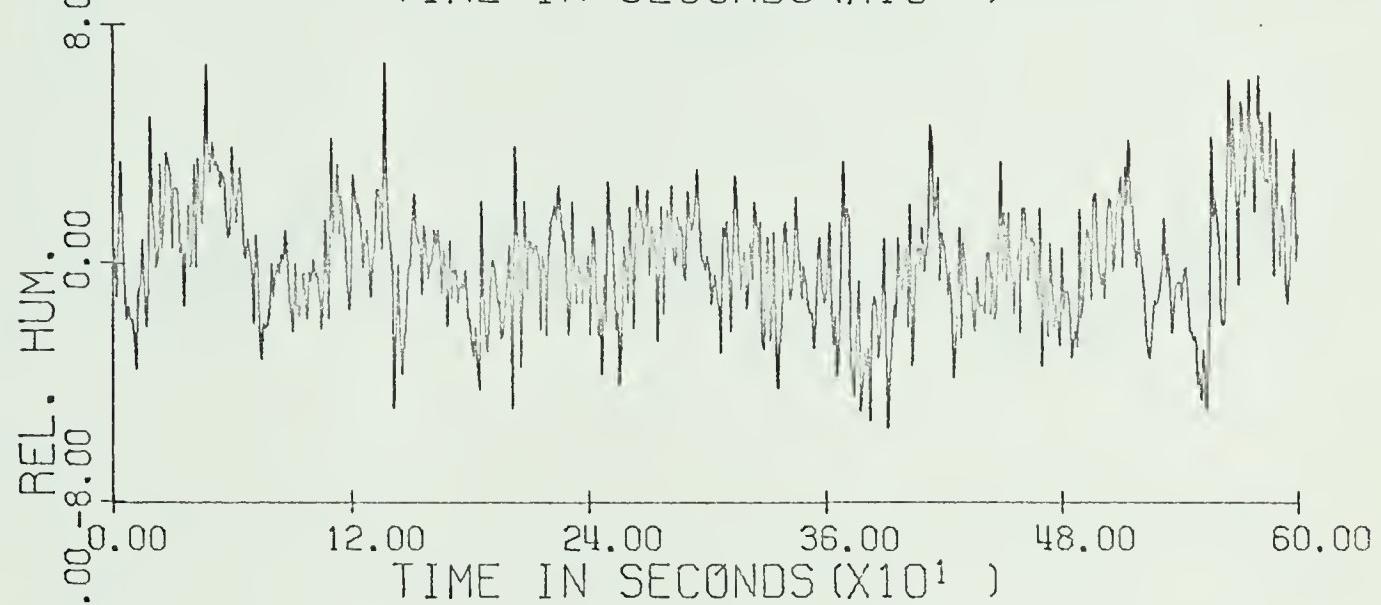
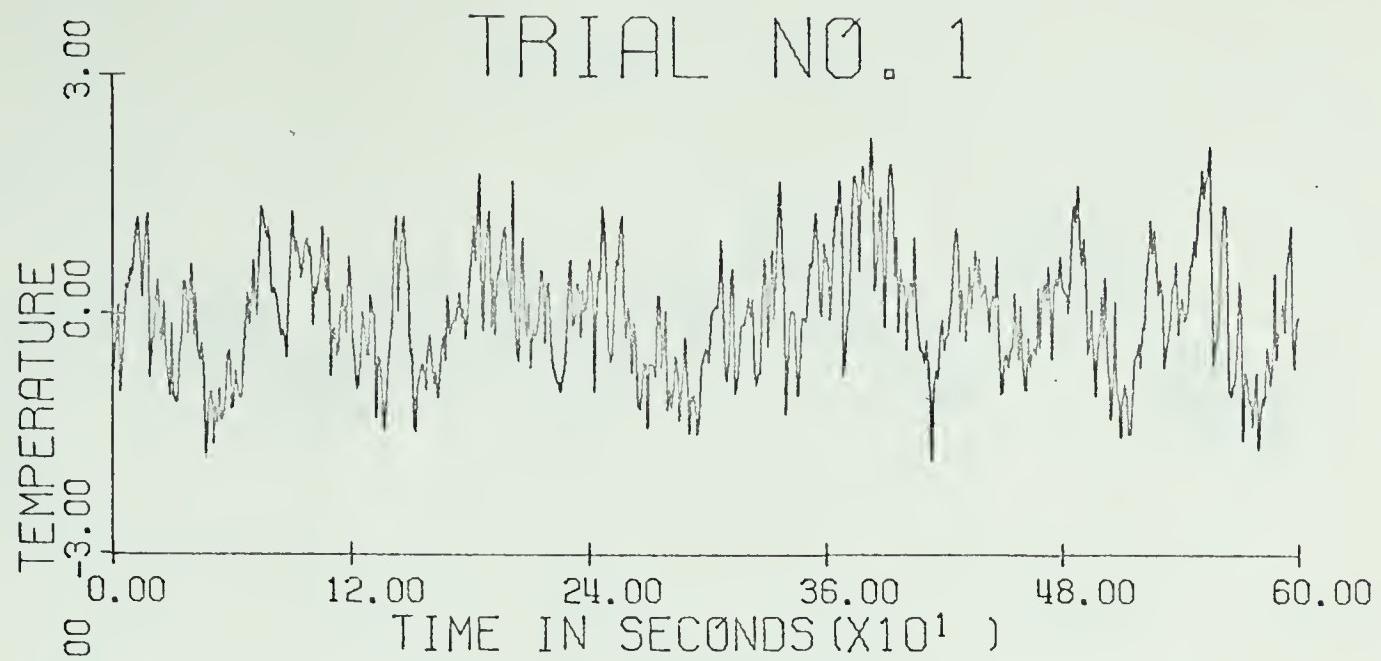
The data used in the spectral and flux calculation are plotted in the following graphs. The units are degrees C for the temperature, per cent for the relative humidity, cm. per sec. for the vertical wind, and g. per kg. for the specific humidity. For ease of plotting only the fluctuations about the mean are given. In all cases, except the vertical wind, any linear trend was removed. The mean values are given below.

Table XII - Means of the Data

No.	T( $^{\circ}$ C)	RH(%)	w(cm/sec)	q(g/kg)
1	18.04	51.26	-4.4	7.04
8	23.89	25.31	-3.6	5.01
11	28.89	20.36	-1.4	5.34
3	26.85	23.32	-2.8	5.54
6	32.32	37.41	-6.8	12.09
9	19.46	26.54	-3.7	4.02
7	20.88	44.79	-5.2	7.45
10	12.28	44.23	-0.2	4.21
13	22.09	29.58	-0.3	5.41
1L	17.70	51.54	-3.7	--
8L	24.56	27.23	-0.4	--
7L	20.97	42.98	-2.8	--
10L	12.22	37.16	-0.1	--

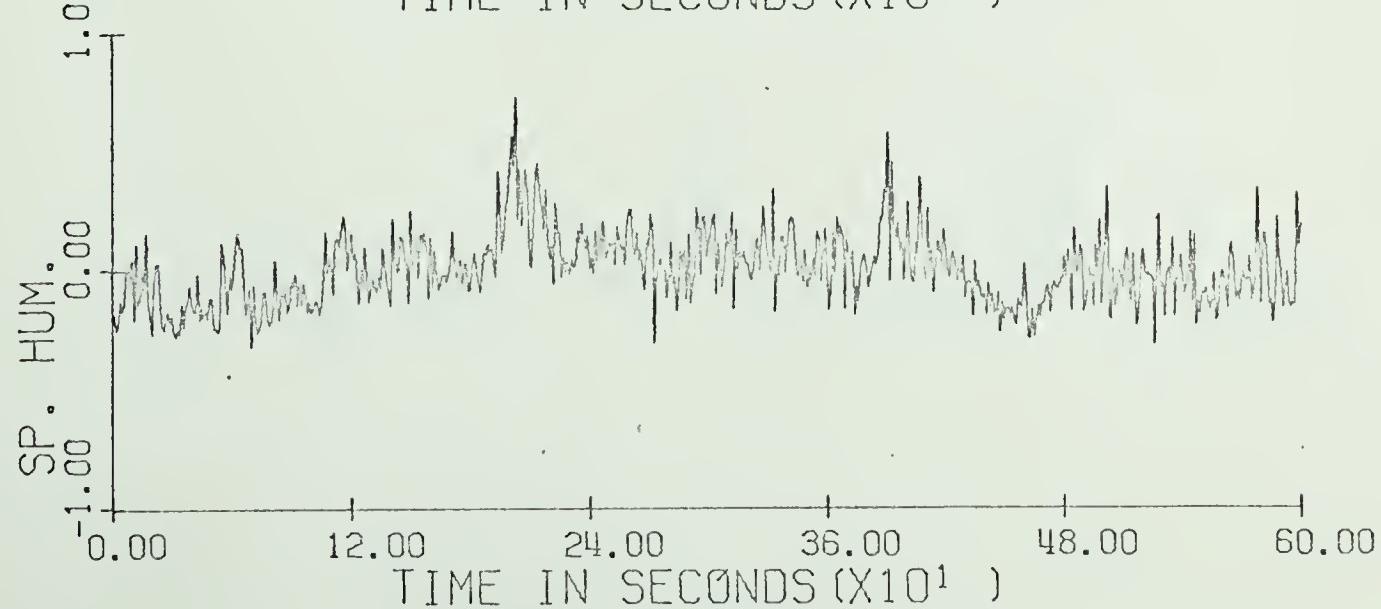
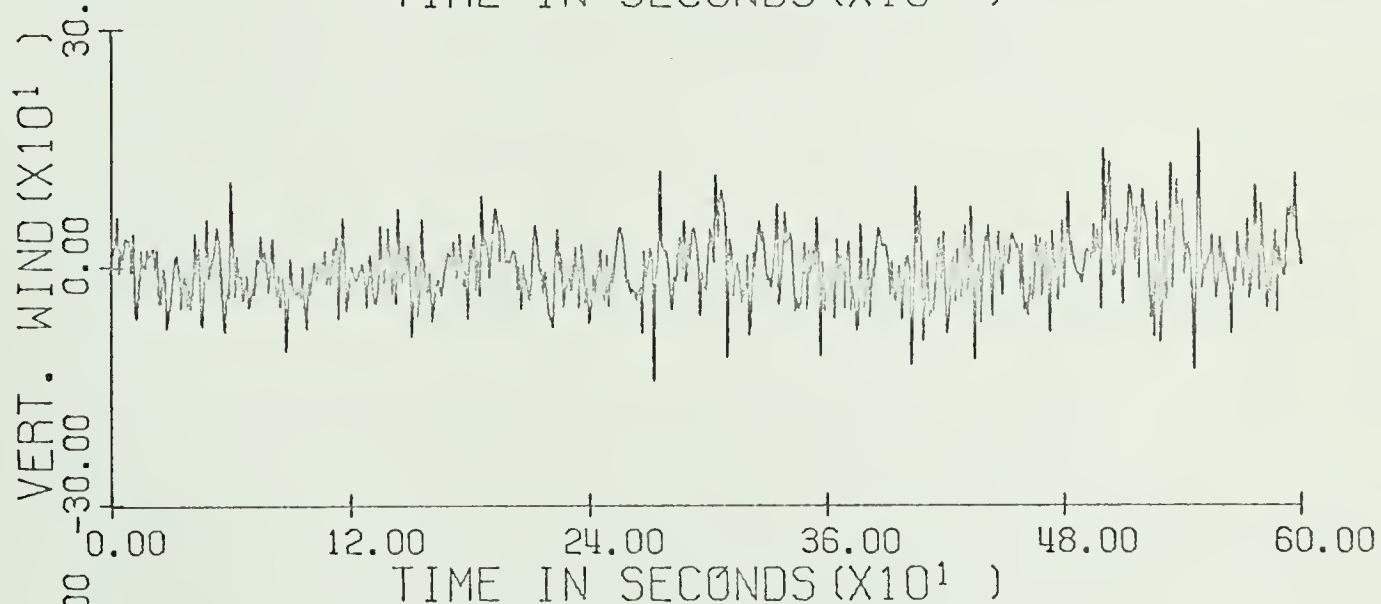
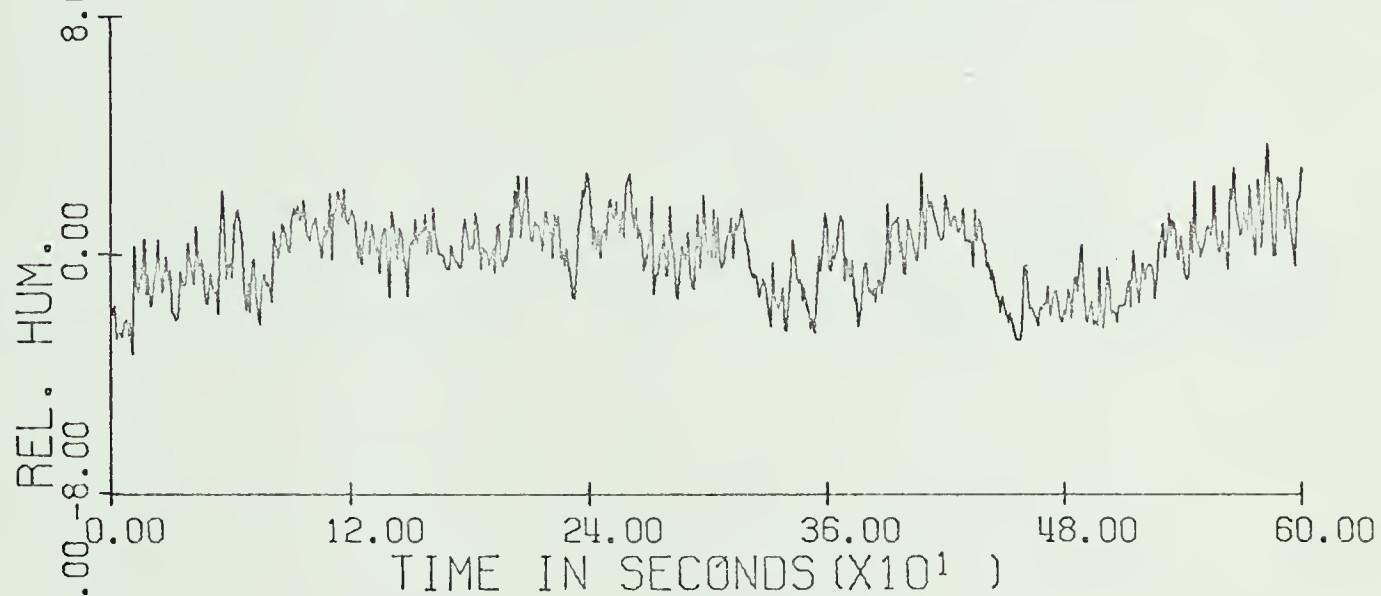
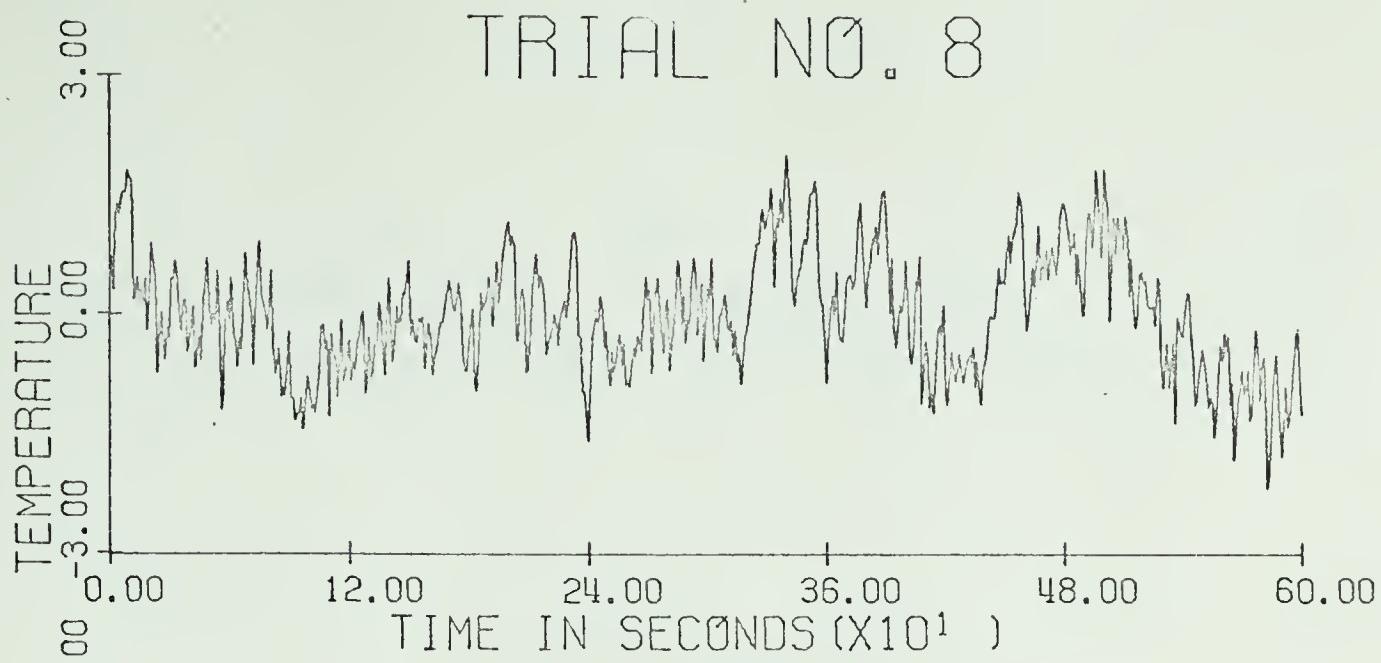


## TRIAL NO. 1



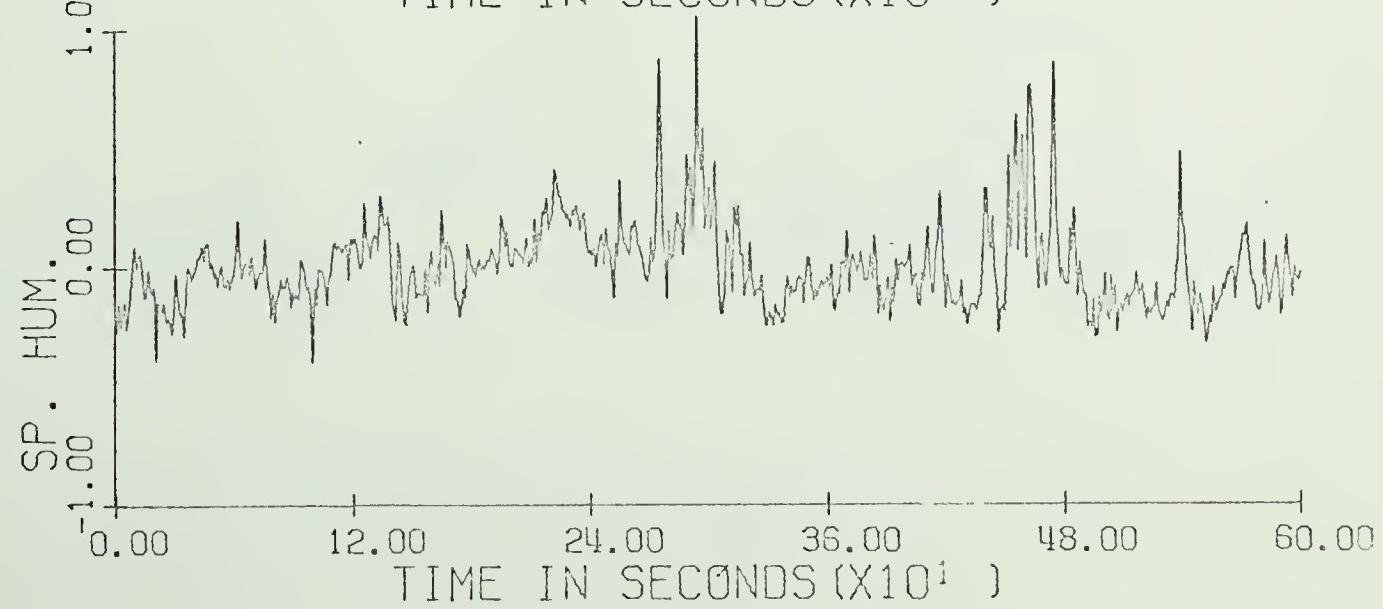
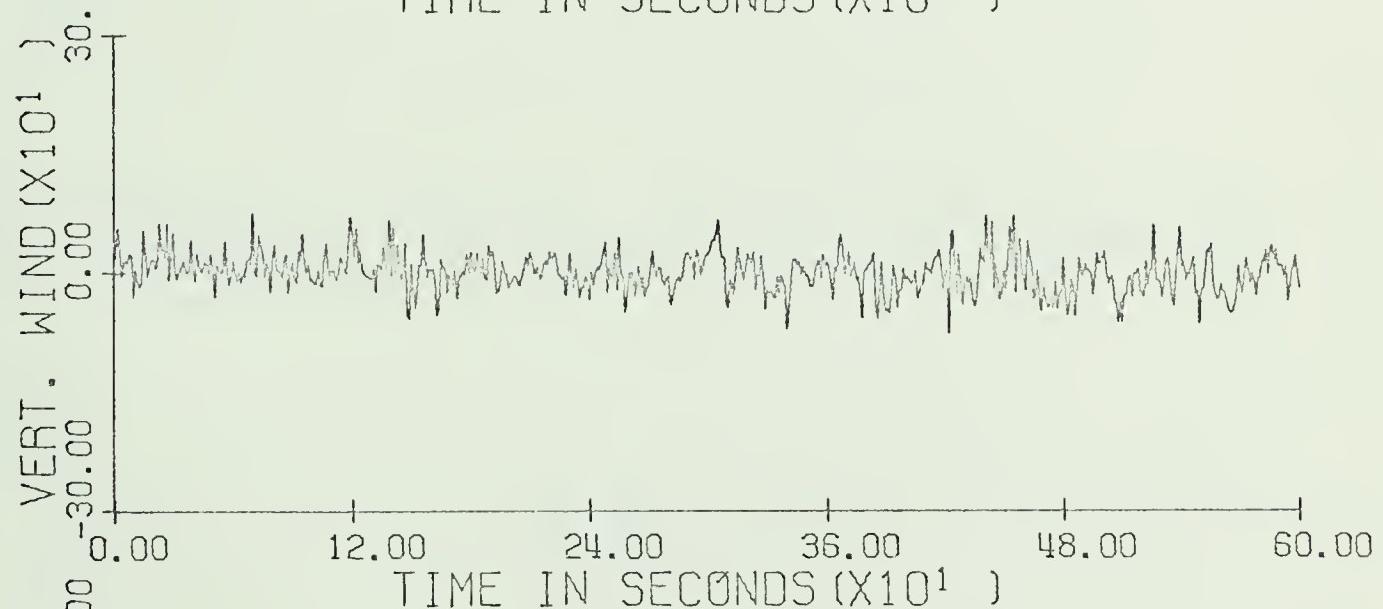
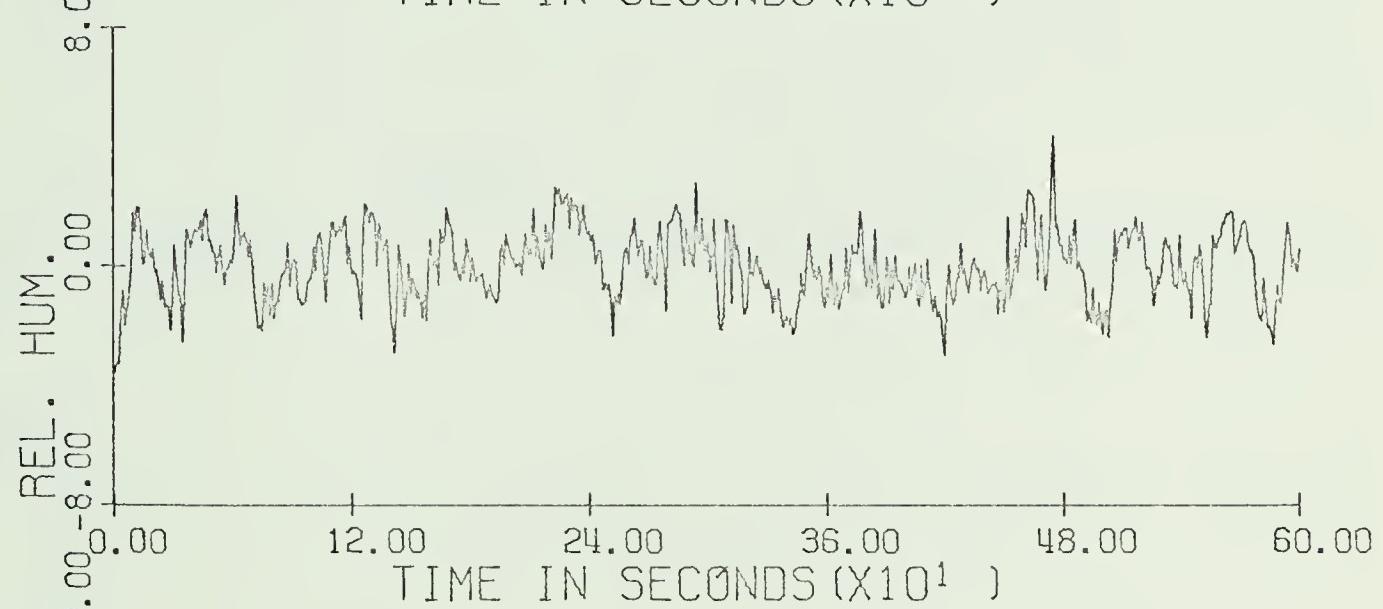
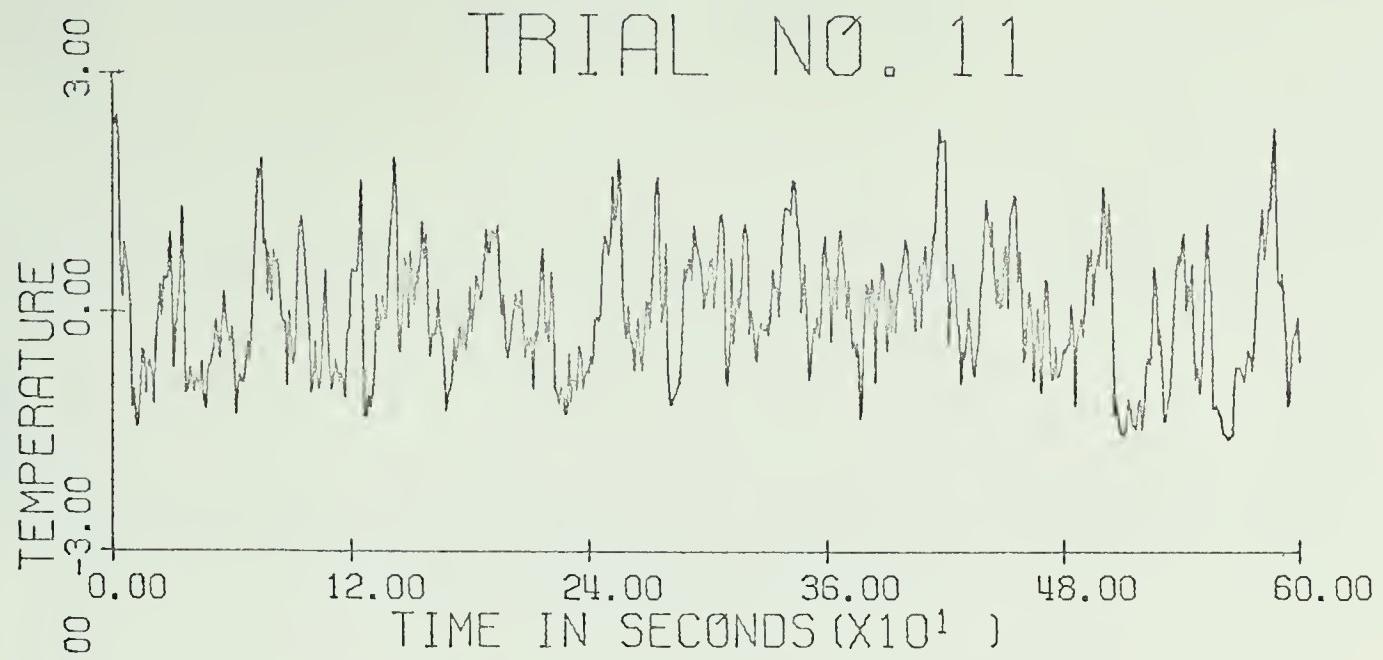


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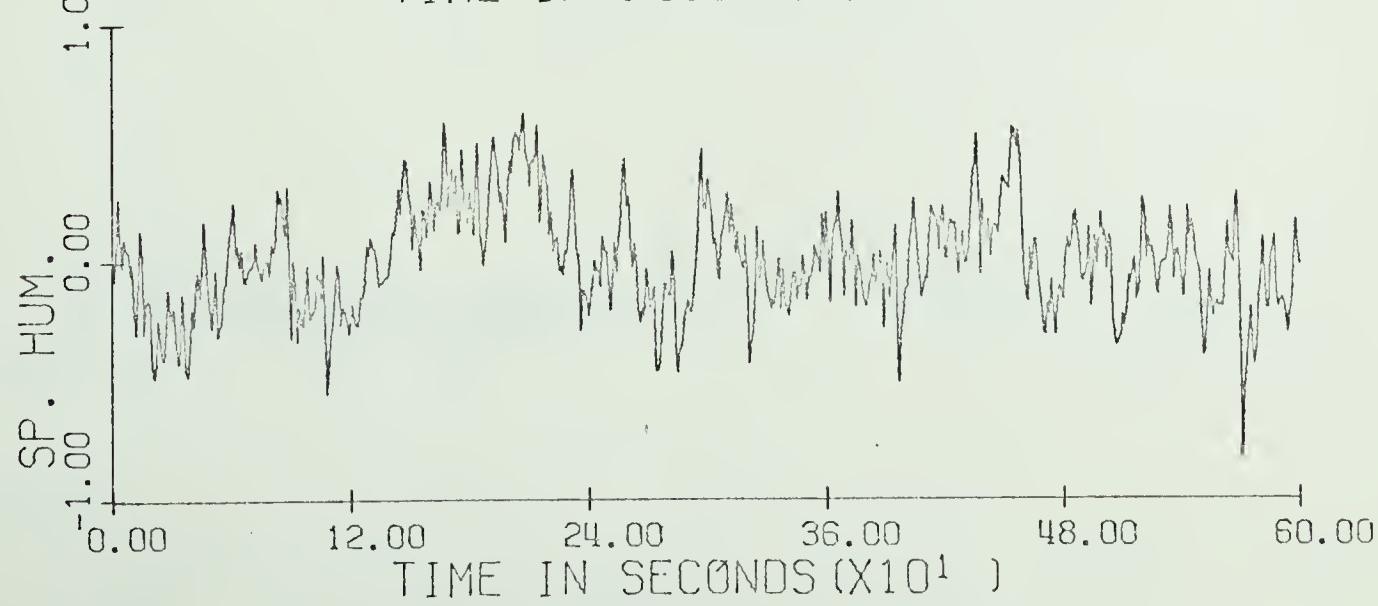
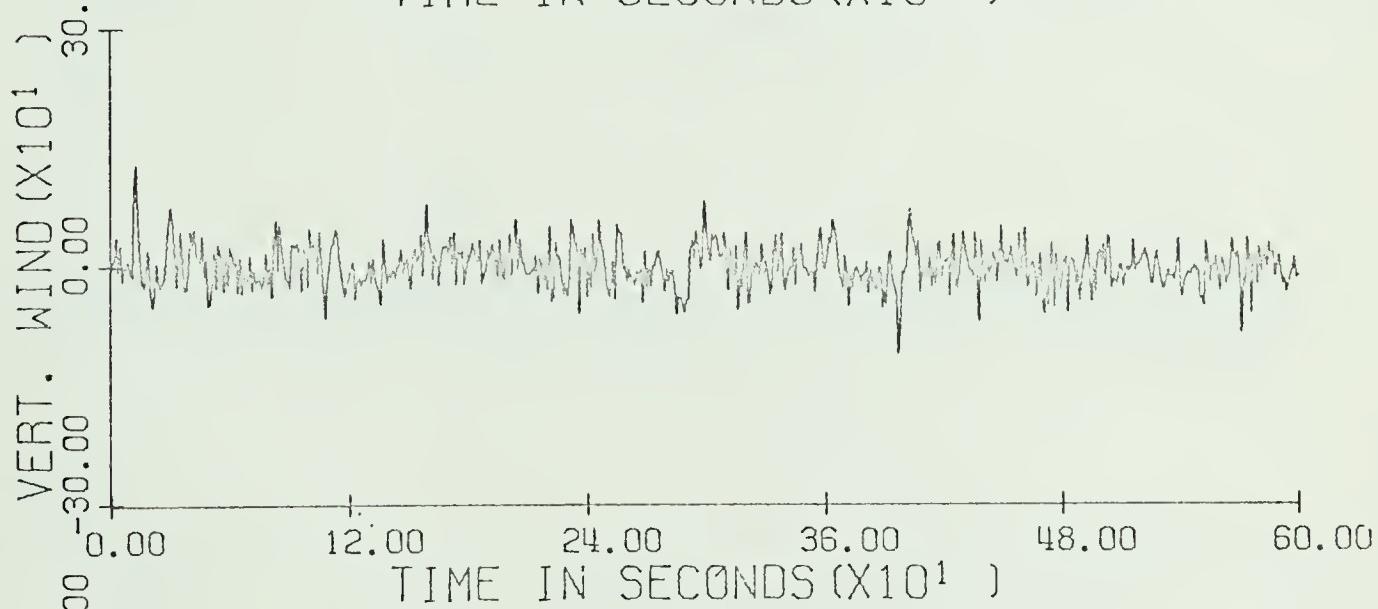
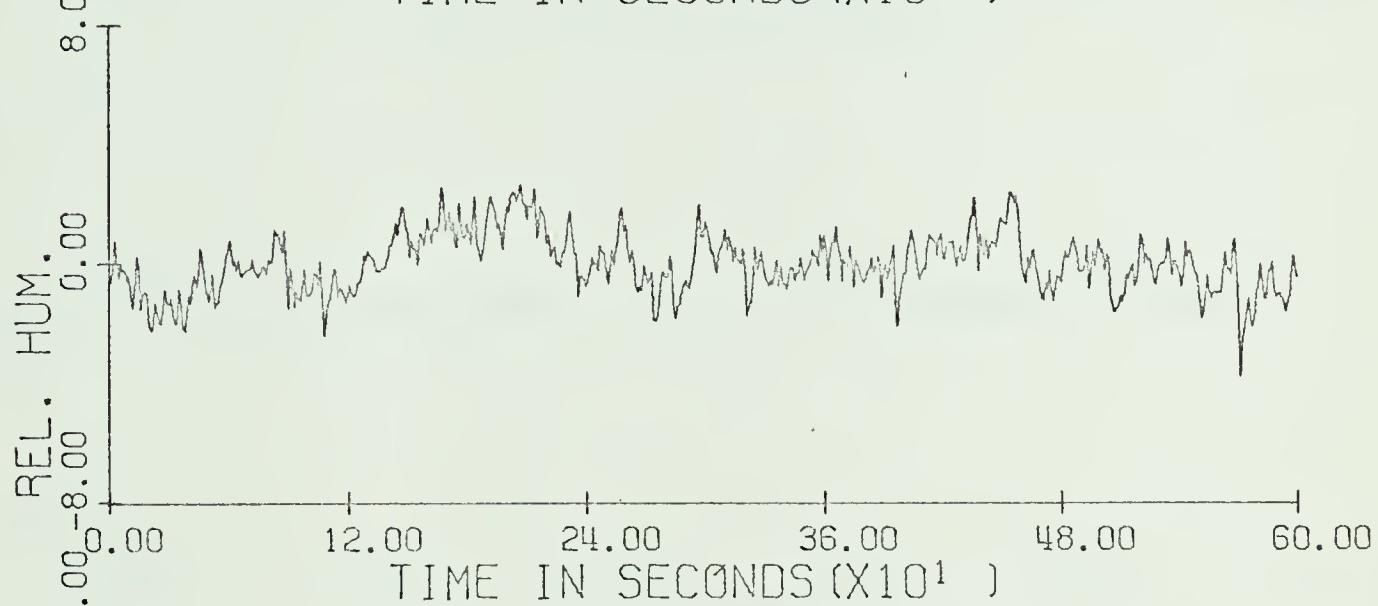


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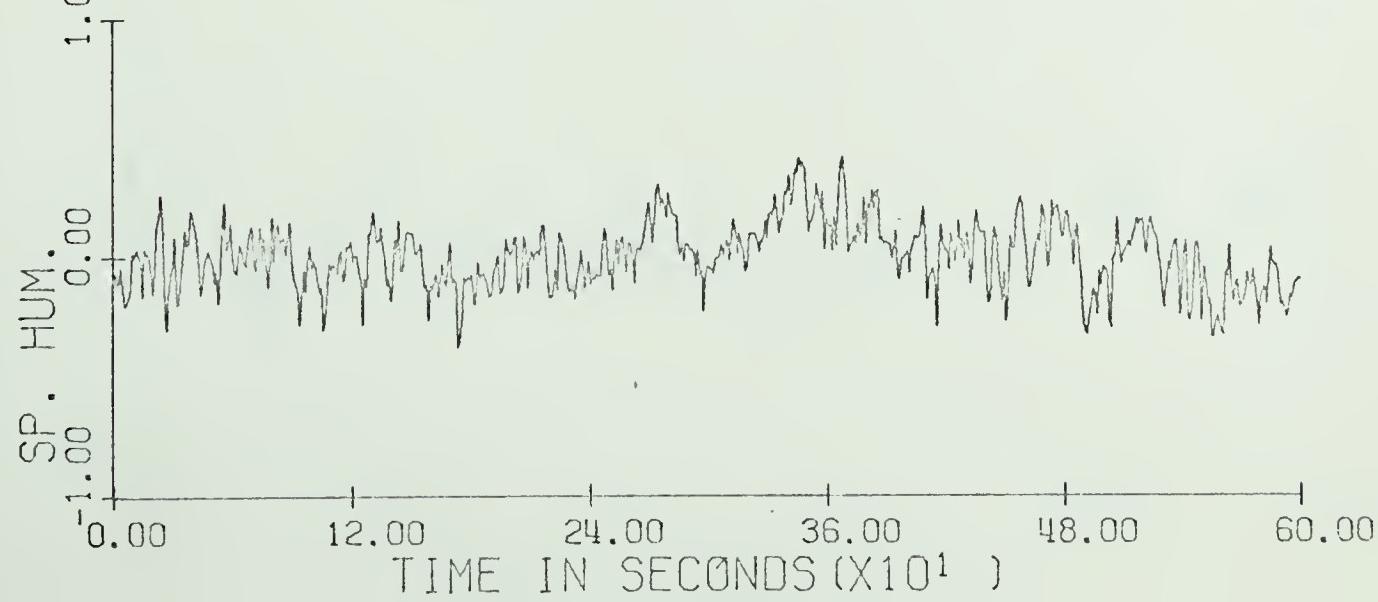
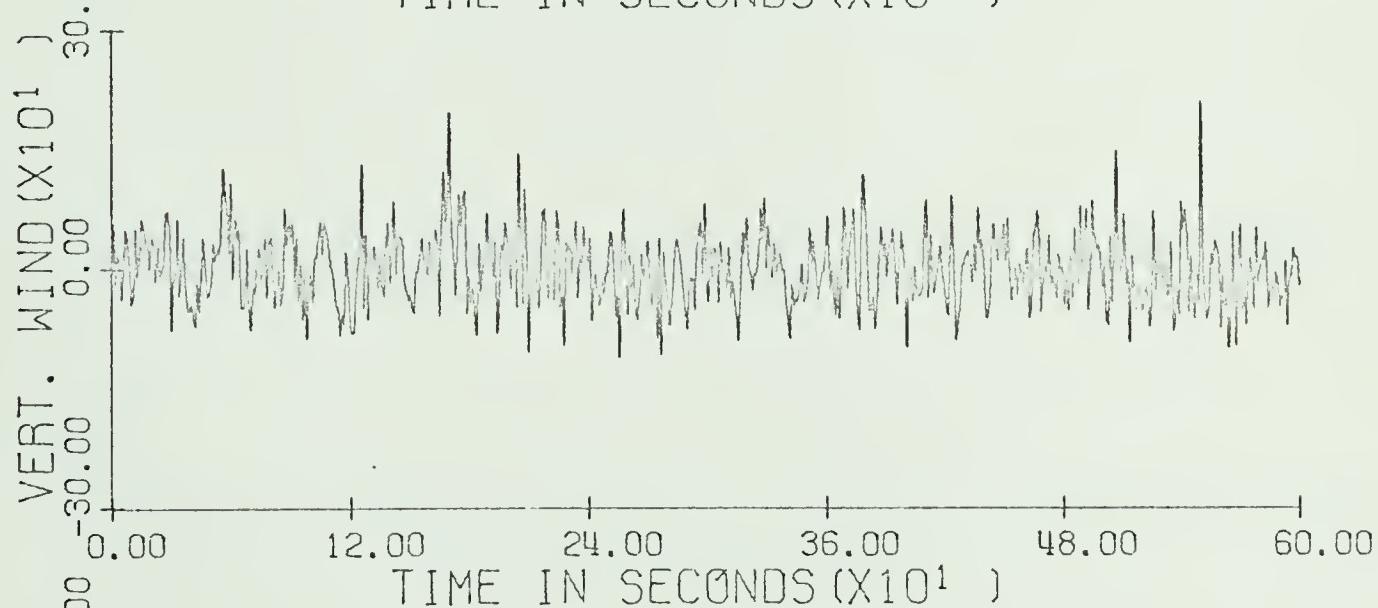
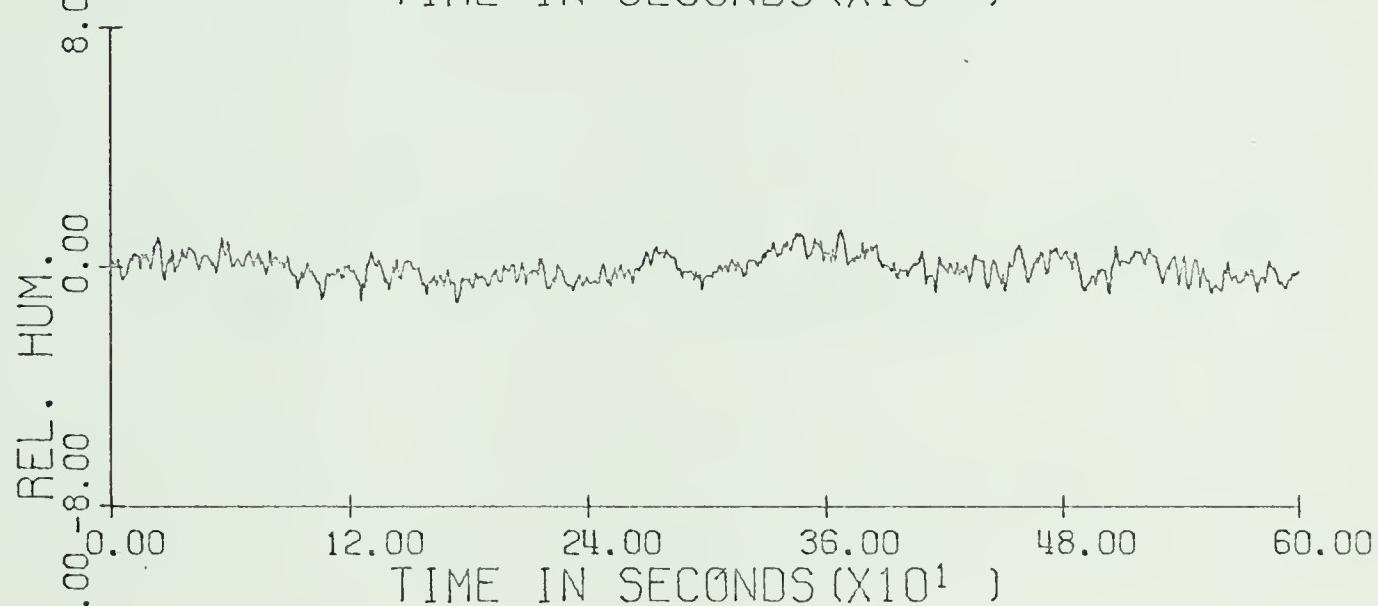
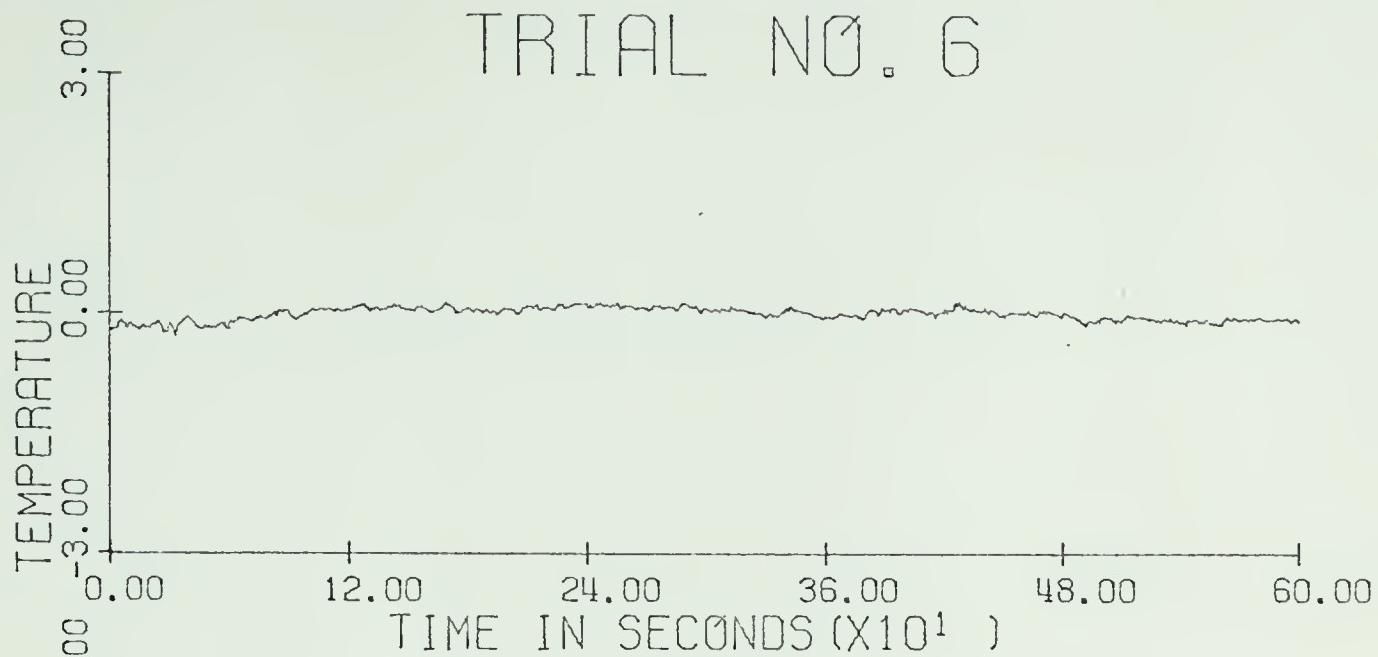


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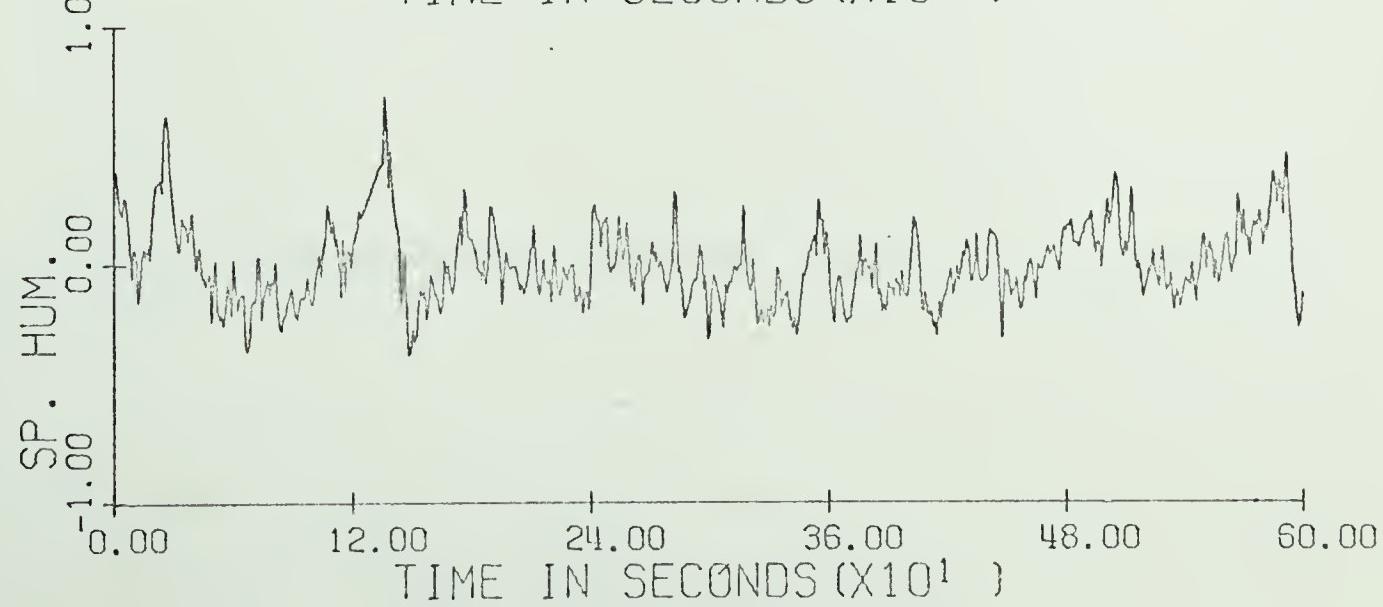
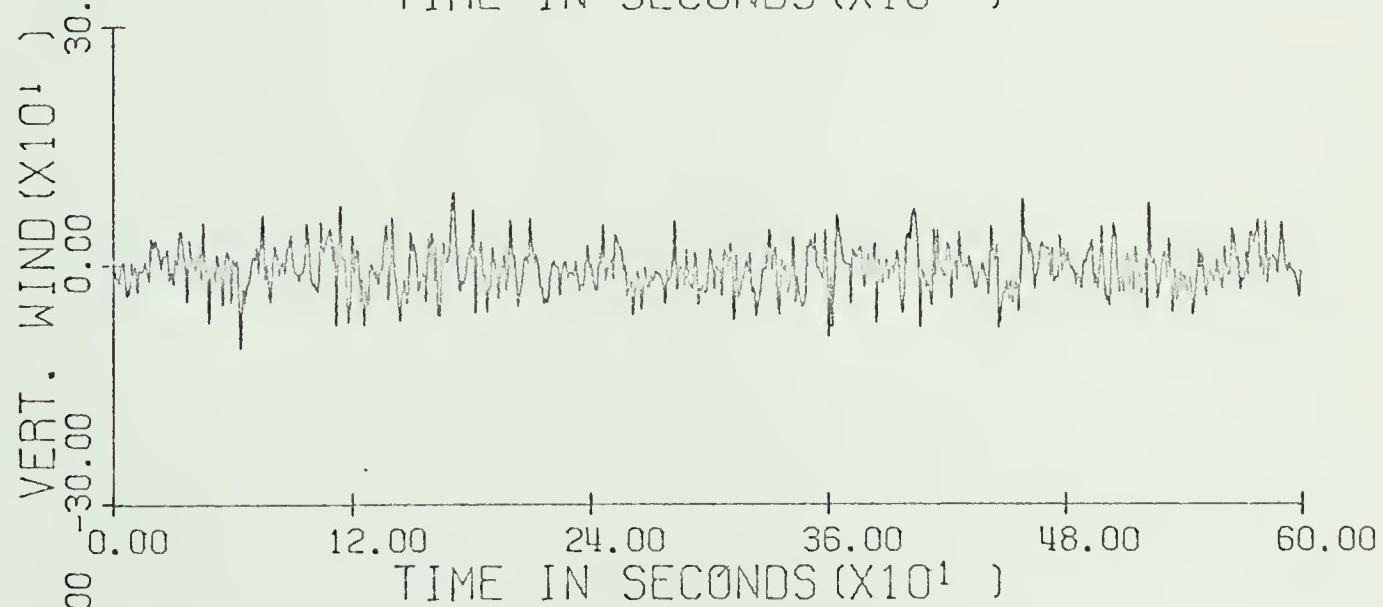
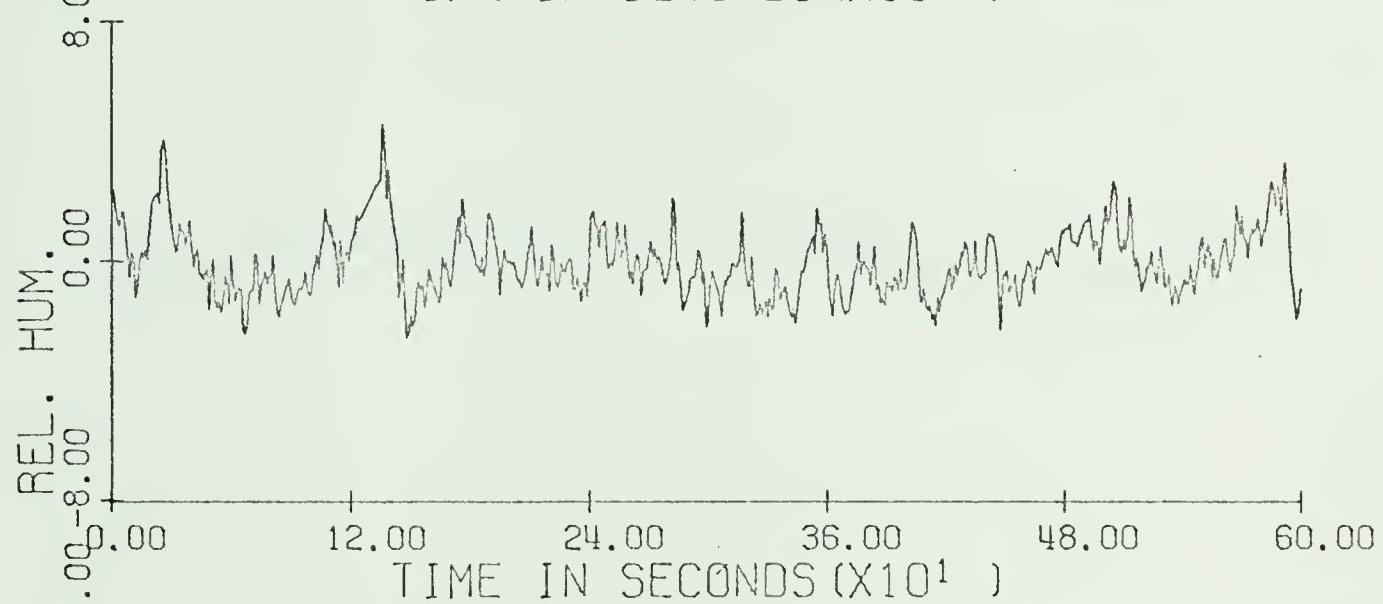
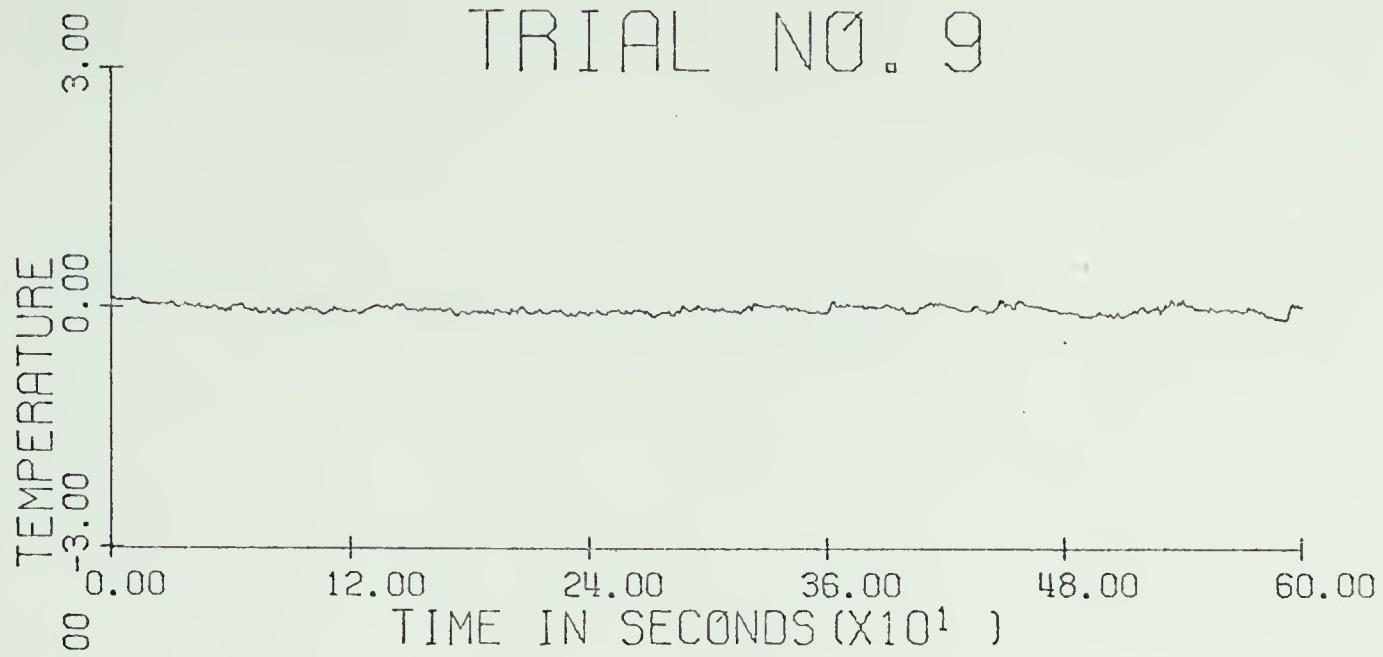


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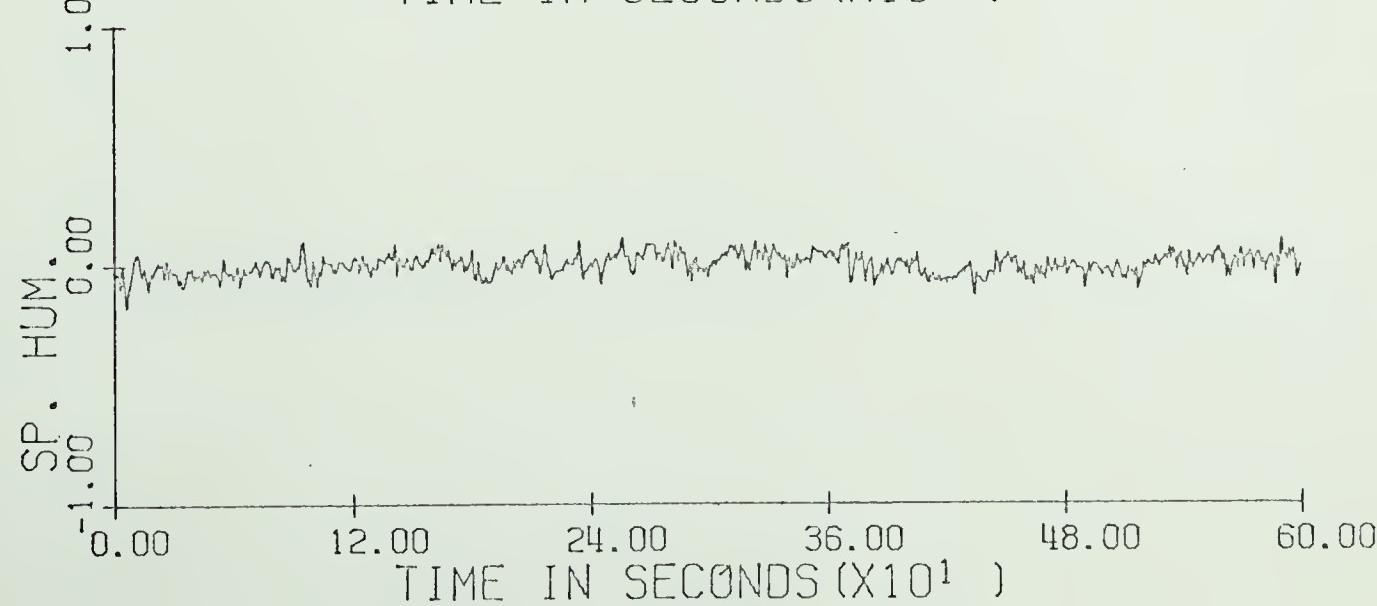
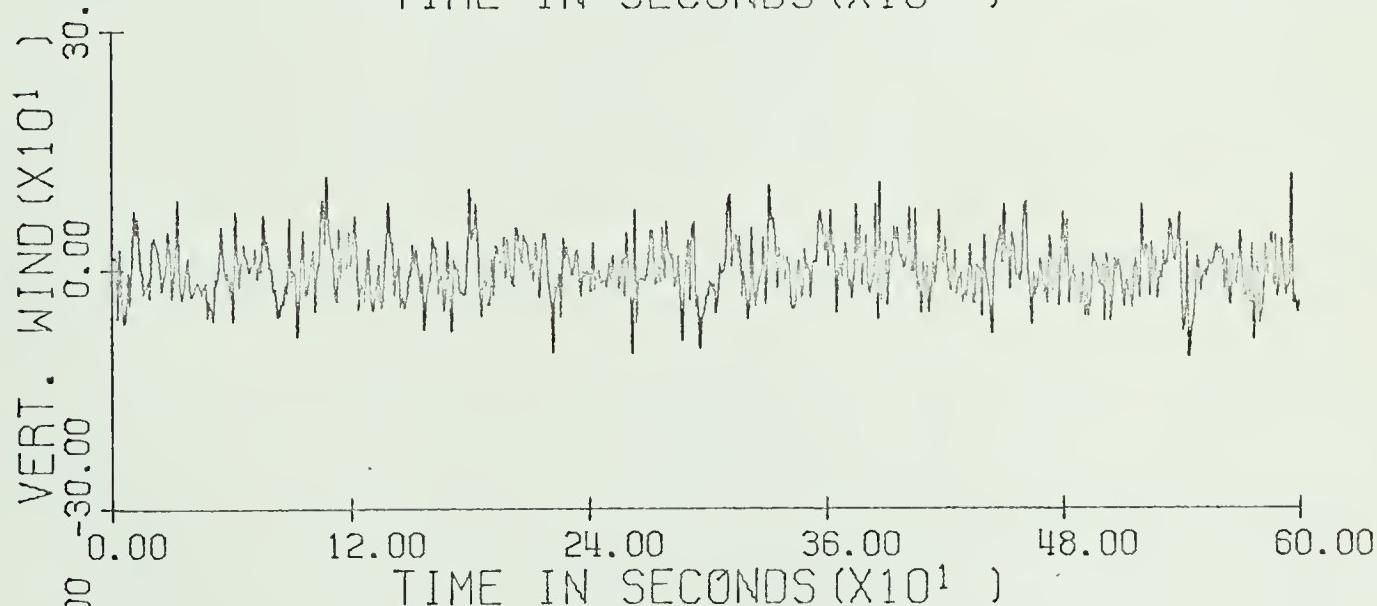
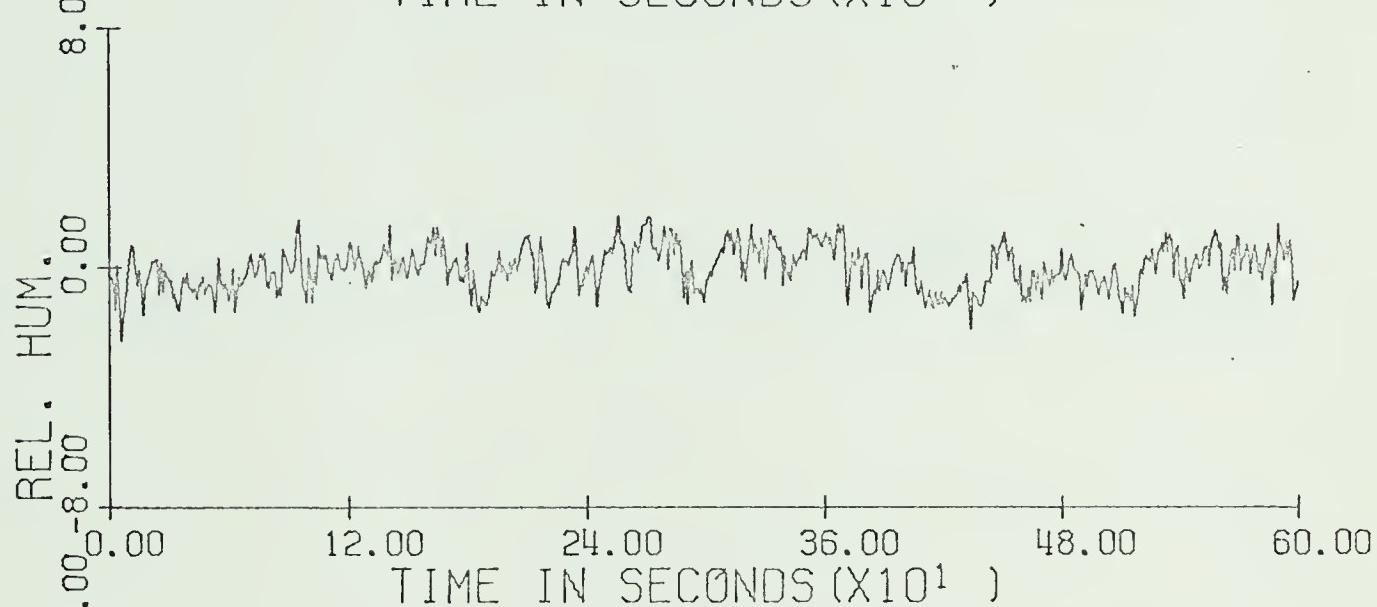
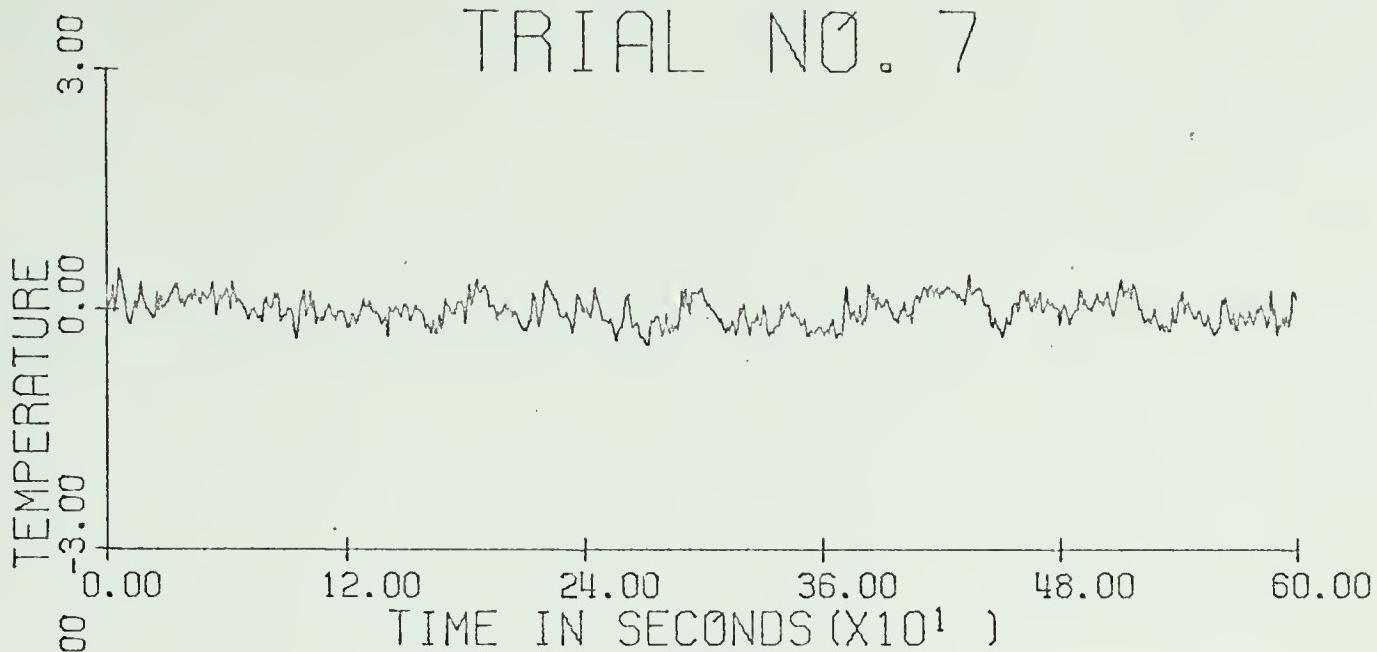


## TRIAL NO. 9



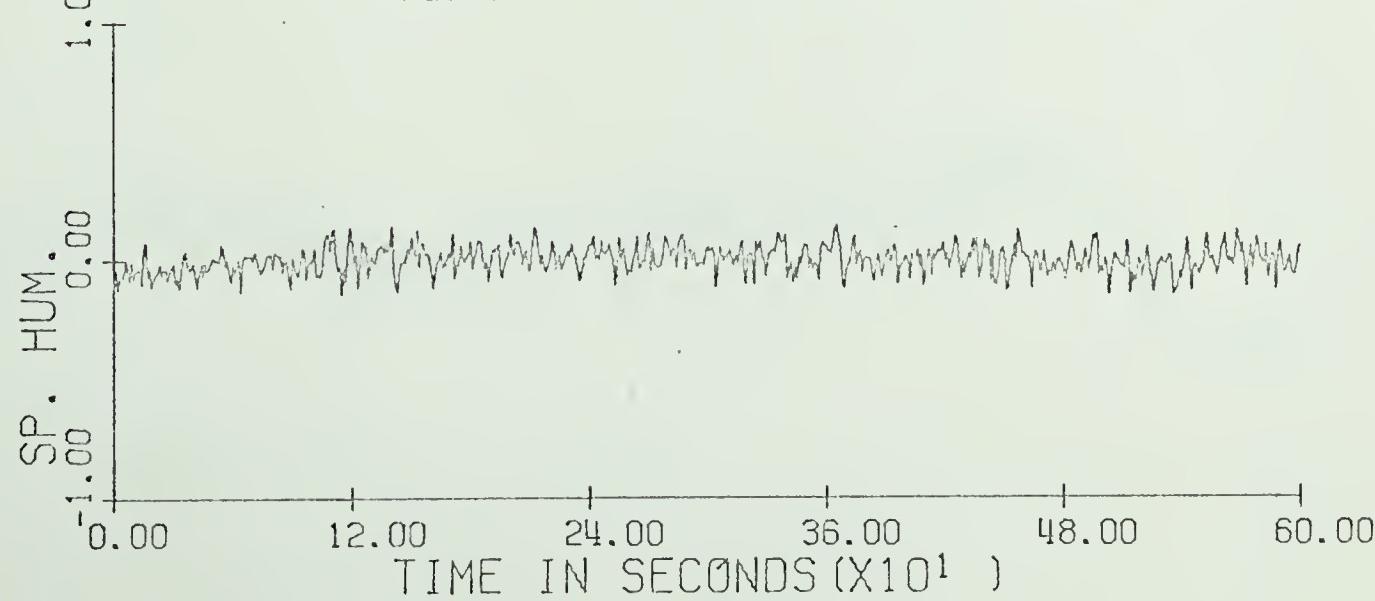
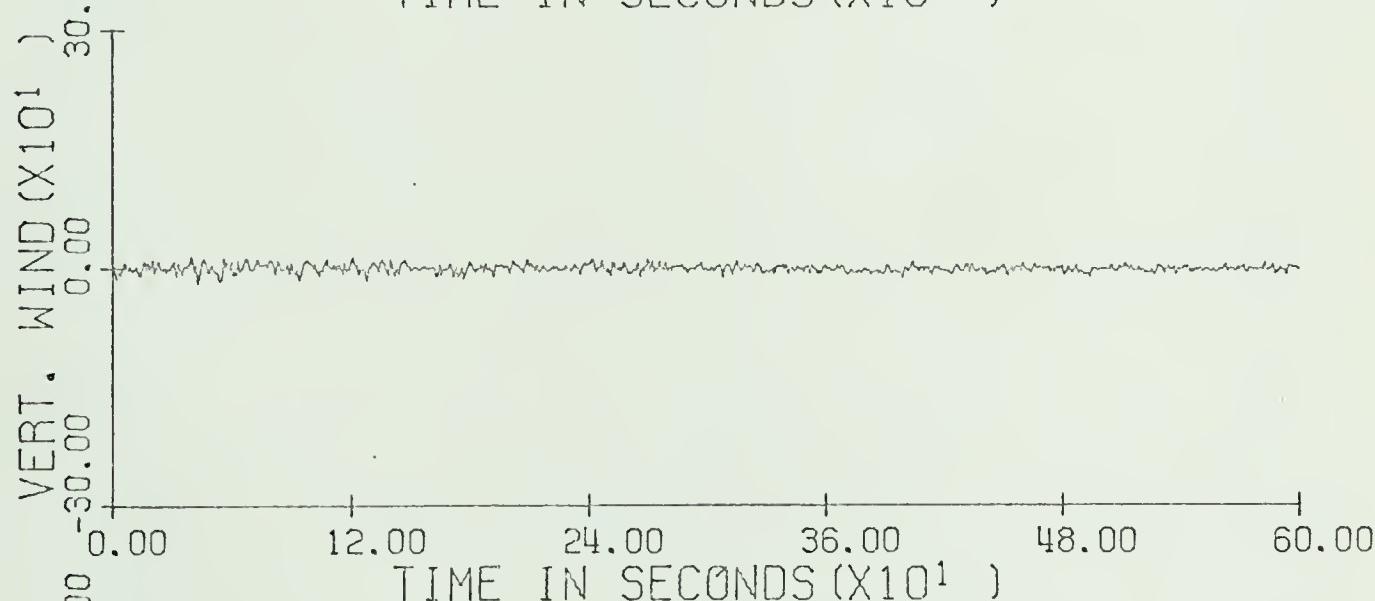
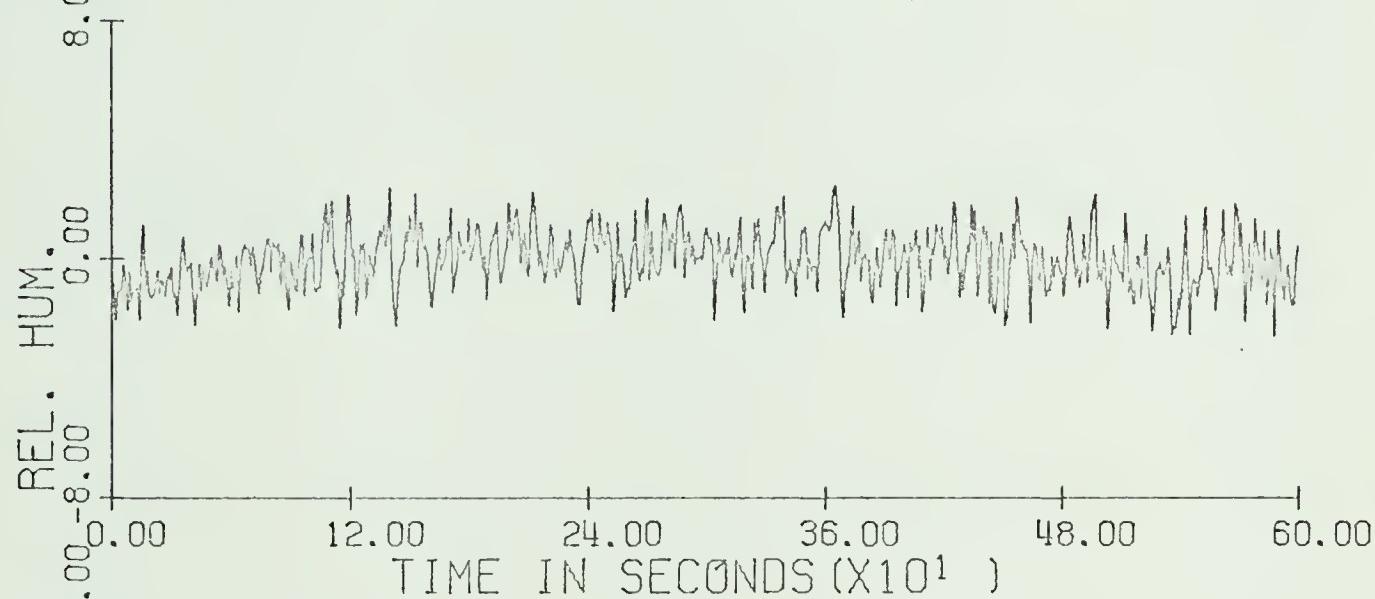
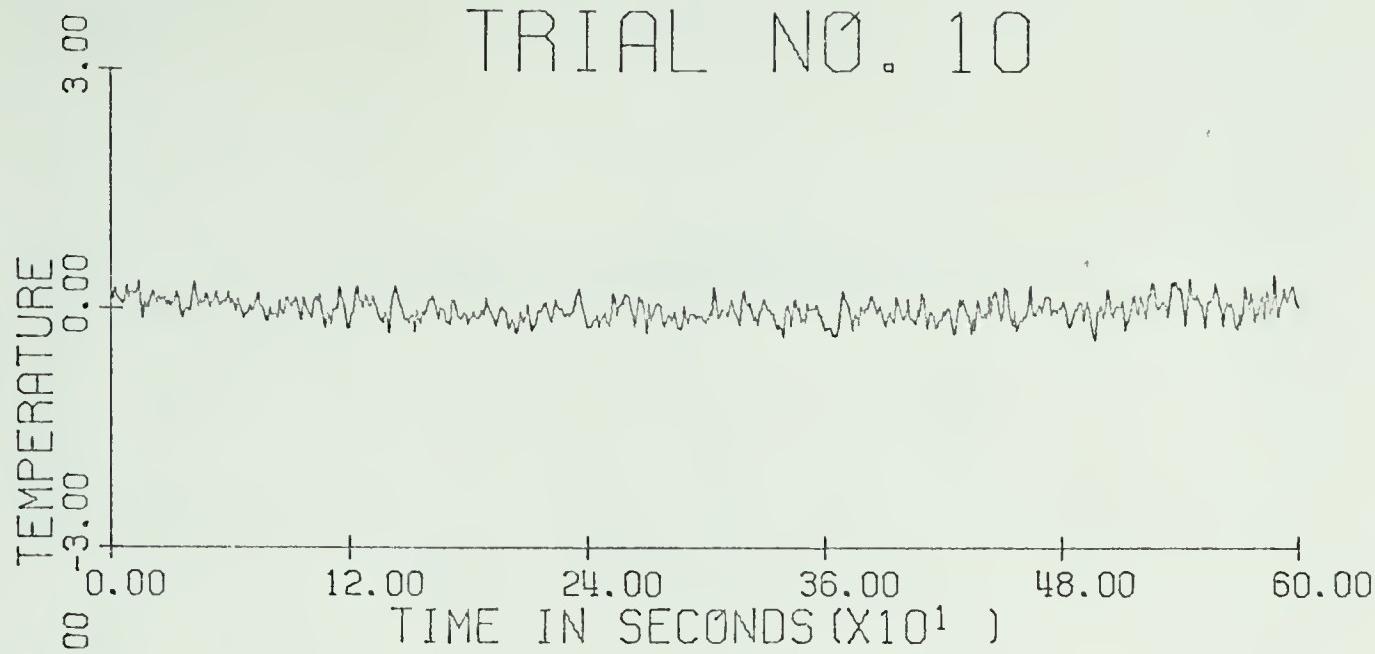


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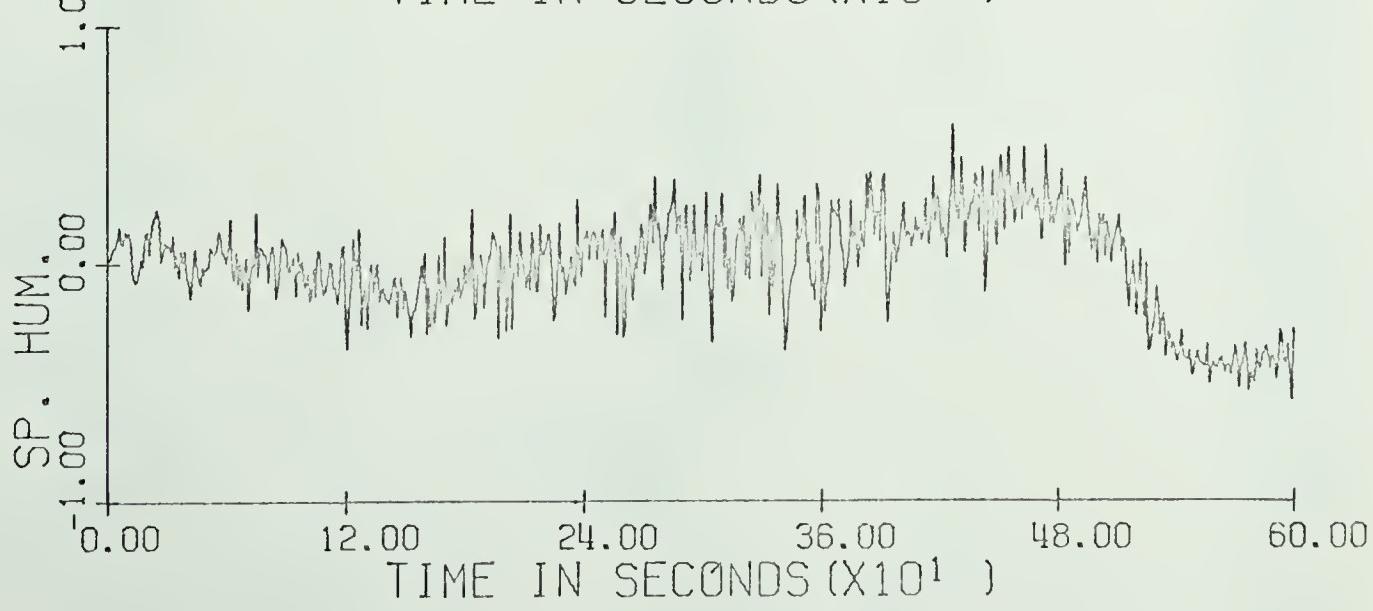
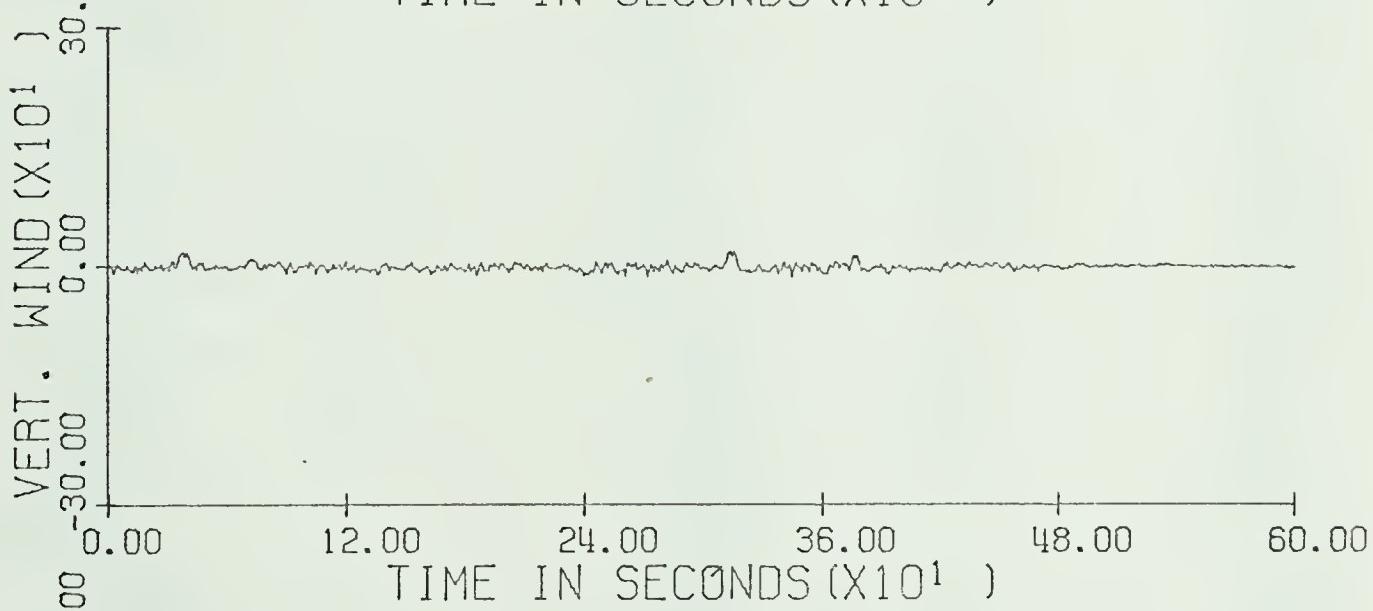
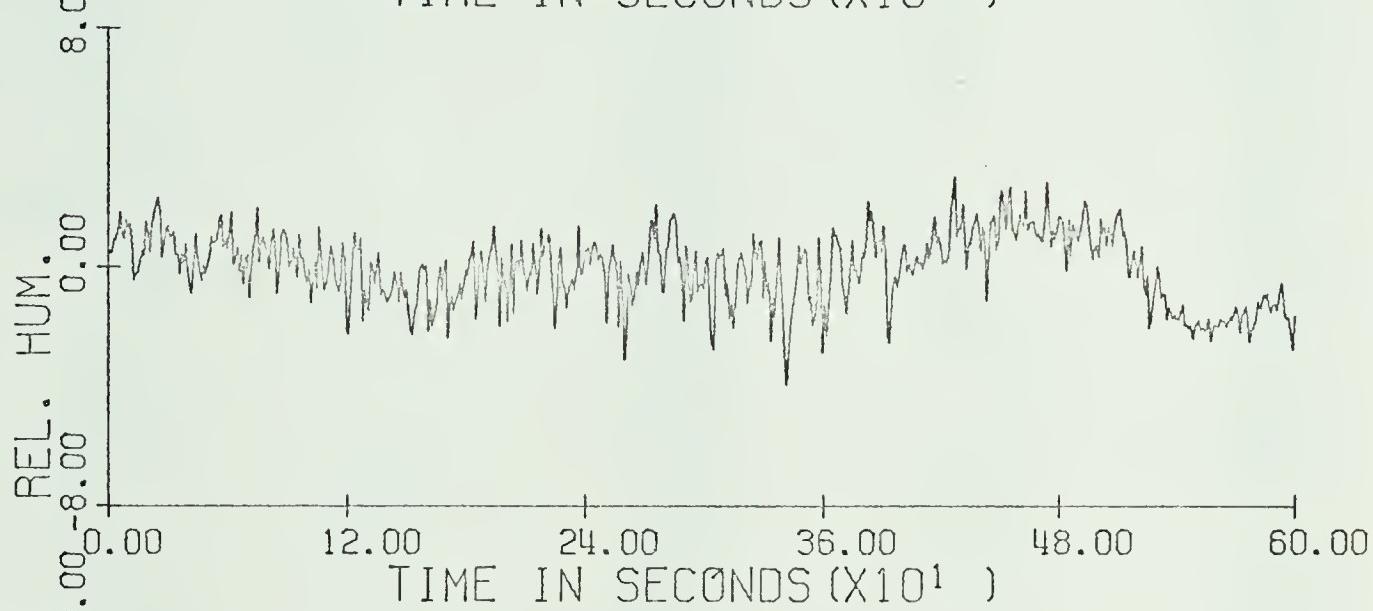
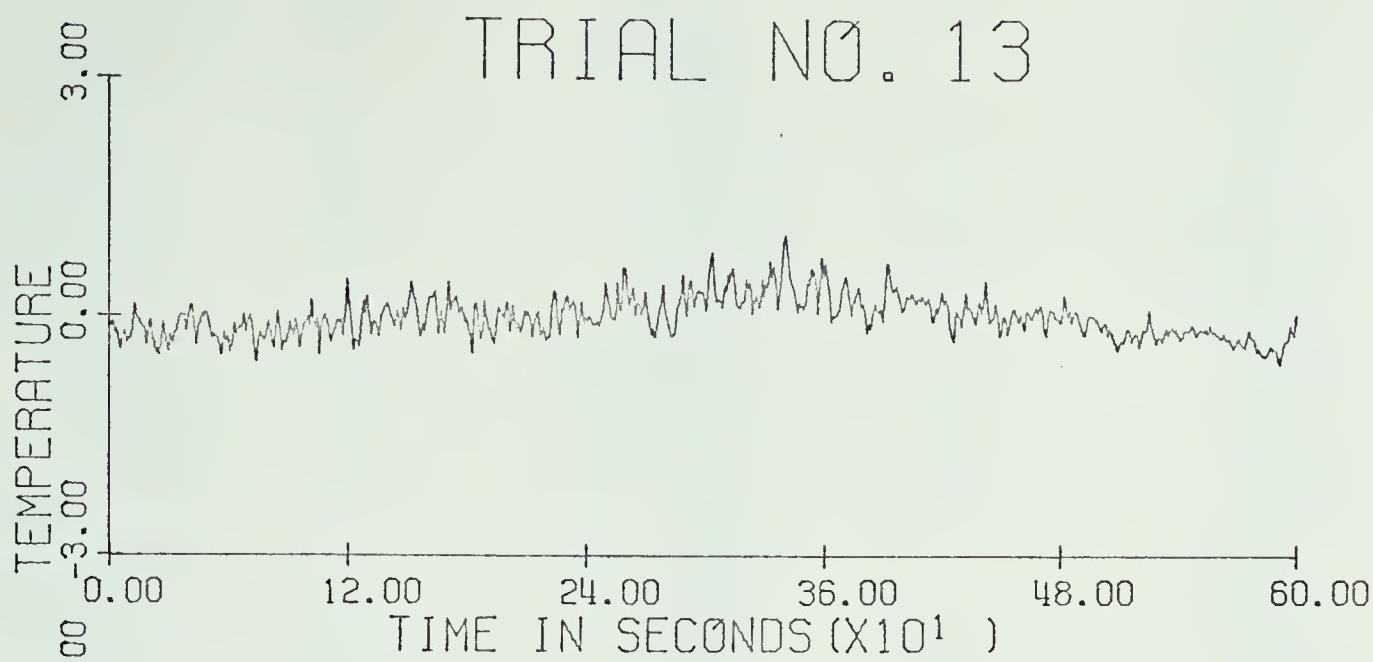


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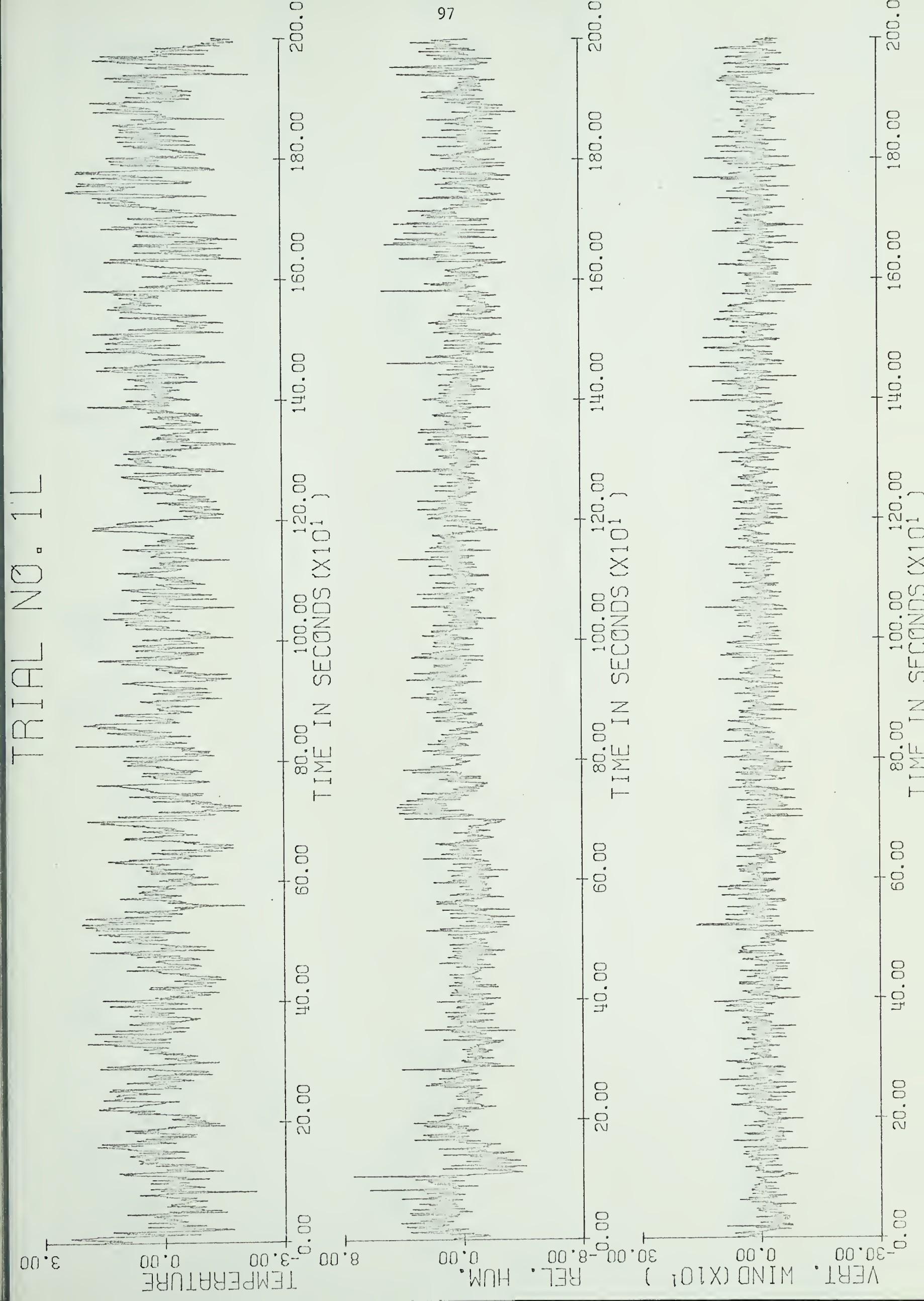


## TRIAL NO. 13





TRIAL NO. 1 L





INITHL NO. 8L

98

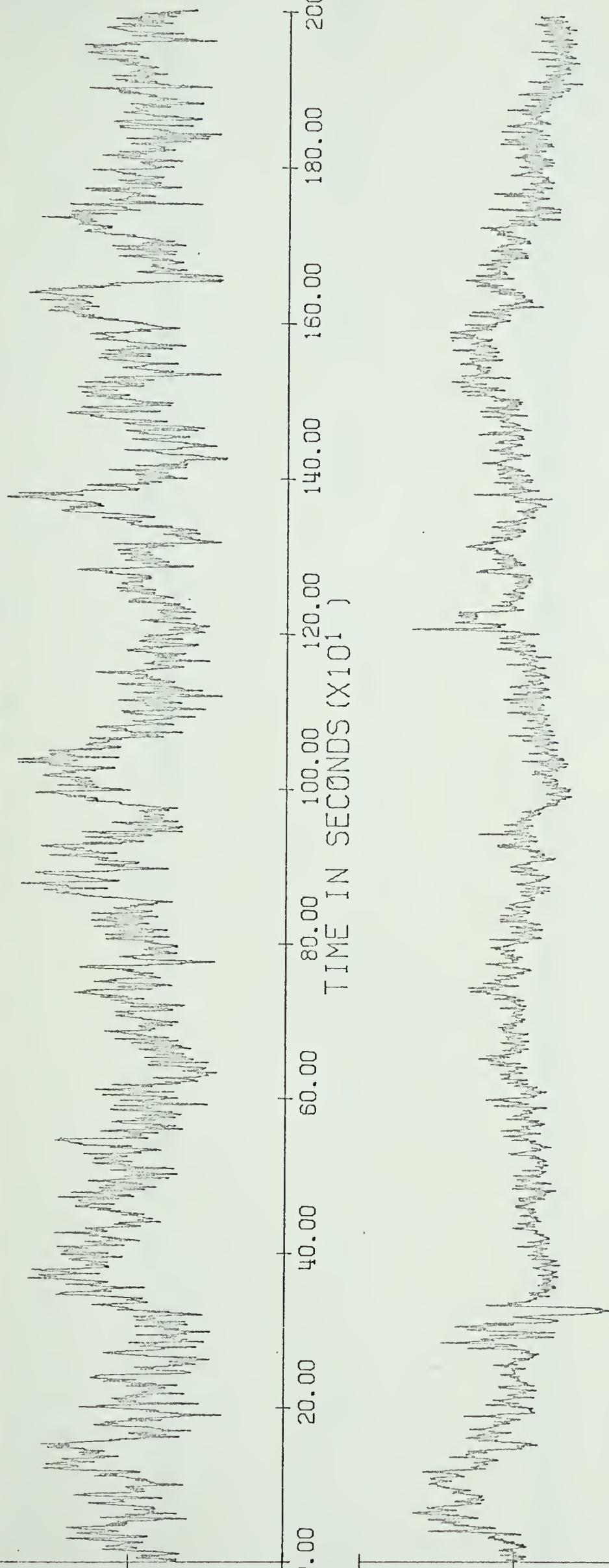
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TEMPERATURE 0.00 8.00 0.00 -8.00

REL. HUM. 0.00 30.00 0.00 -30.00

0.00 20.00 40.00 60.00 80.00 100.00 120.00 140.00 160.00 180.00 200.00

TIME IN SECONDS ( $\times 10^1$ )



0.00 20.00 40.00 60.00 80.00 100.00 120.00 140.00 160.00 180.00 200.00

TIME IN SECONDS ( $\times 10^1$ )

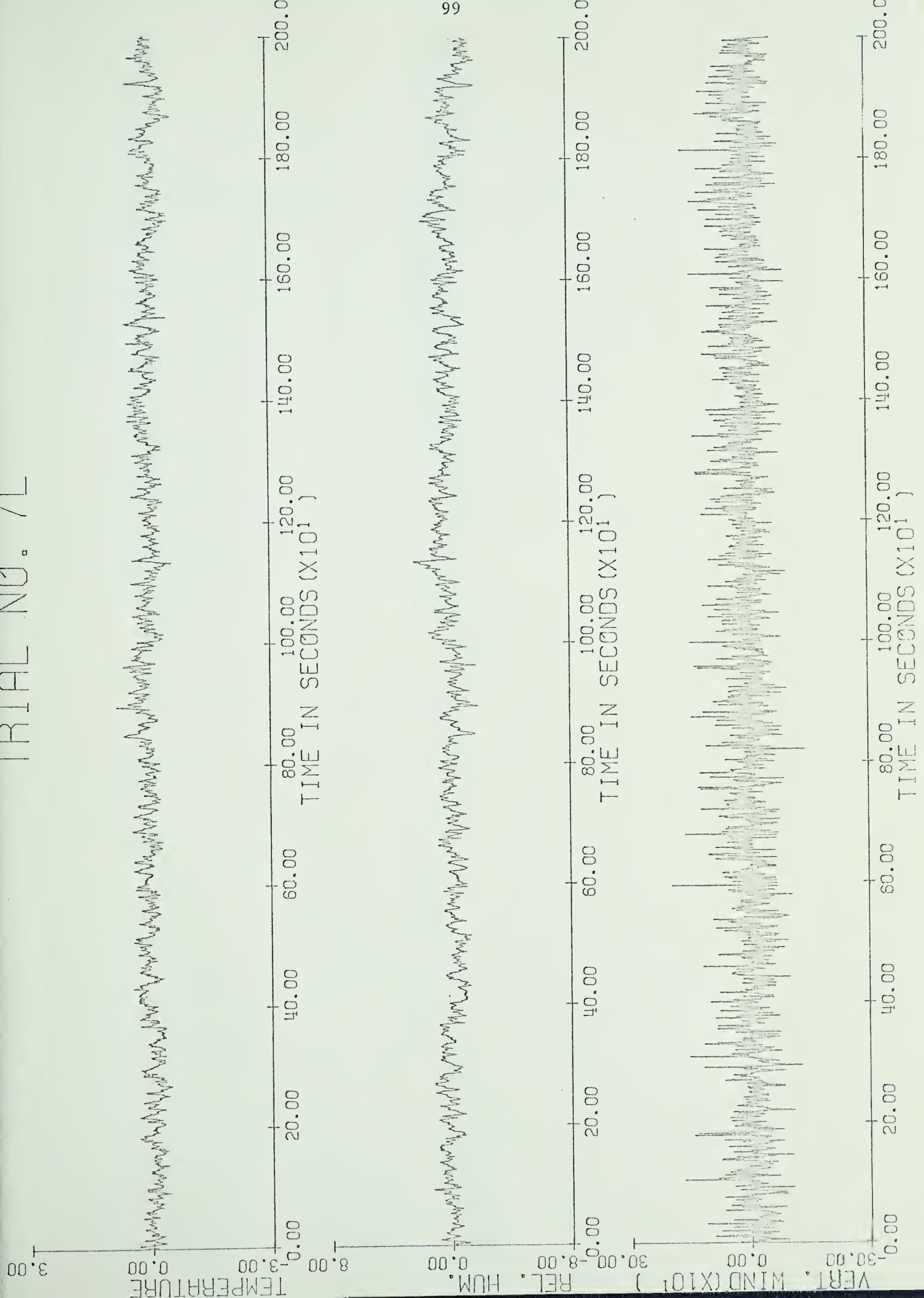


0.00 20.00 40.00 60.00 80.00 100.00 120.00 140.00 160.00 180.00 200.00

TIME IN SECONDS ( $\times 10^1$ )

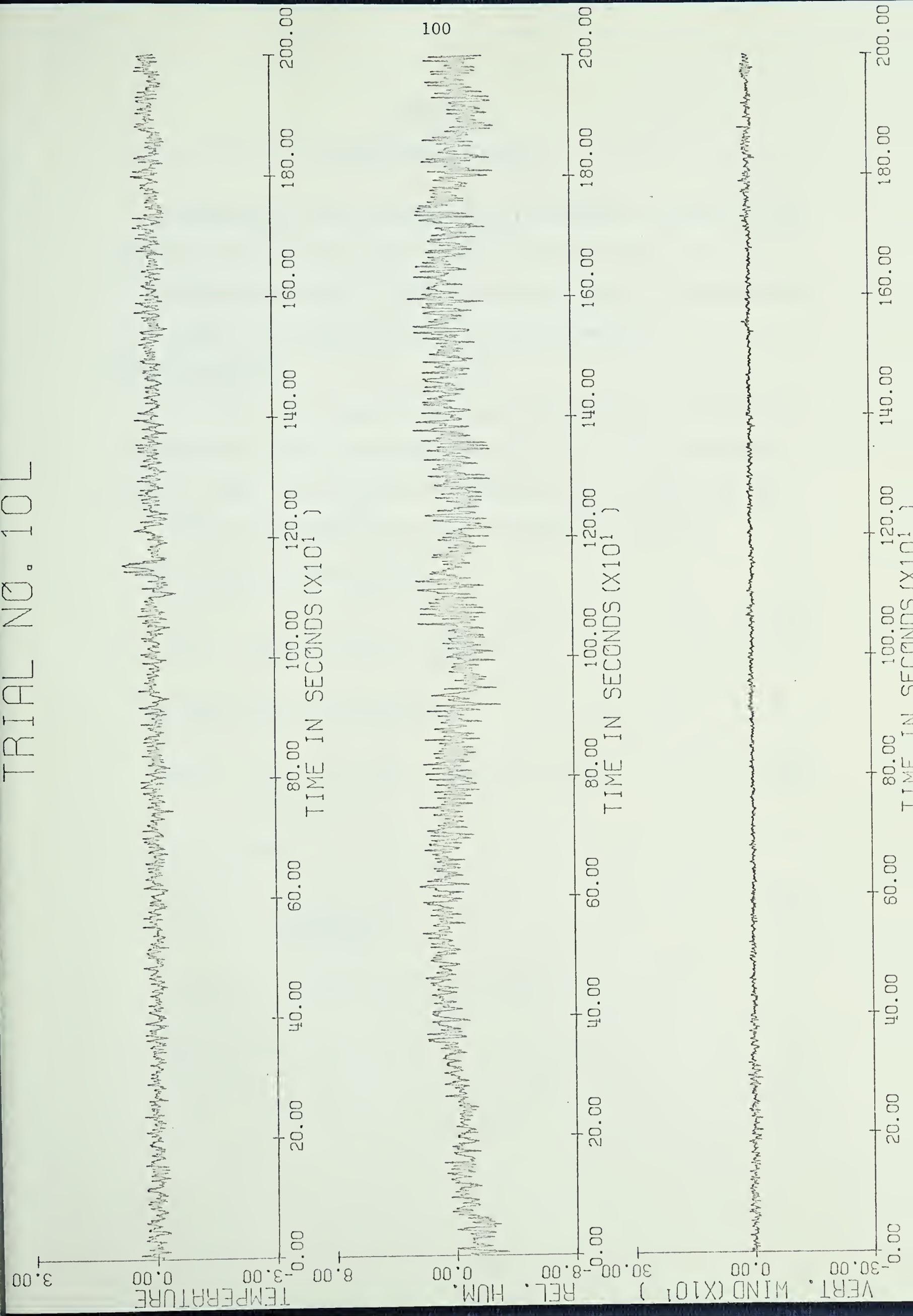


TRIAL NO. 7 L





TRIAL NO. 10 L





APPENDIX D  
COMPUTATIONAL FORMULAE

Covariances, correlations, spectra, coherence, and phaselead are calculated from the time series having the form  $x_i$  and  $y_i$ ,  $i = 1, 2, 3\dots N$ .  $N$  is the total number of points in the data series. Also calculated are the mean, the standard deviation, and the skewness.

Spectral Formulae

The time series should have their means removed. The equations are evaluated for lag,  $n$ , having values  $0, 1, 2\dots M$ .  $M$  is the maximum number of lags. The summations of Equations (D.1) to (D.5) are for  $i = 1$  to  $N-n$  and for Equations (D.10) to (D.13)  $k = 1$  to  $M-1$ .

Primary record autocovariance:

$$A(n) = \frac{1}{N-n} \sum x_i x_{i+n} - \frac{1}{(N-n)^2} \sum x_i \sum x_{i+n} \quad (D.1)$$

Secondary record autocovariance:

$$B(n) = \frac{1}{N-n} \sum x_i x_{i+n} - \frac{1}{(N-n)^2} \sum x_i \sum x_{i+n} \quad (D.2)$$

Autocorrelation:

$$\text{COR}(n) = \frac{(N-n)^2 A(n)}{\sqrt{S_x S_{xn}}} \quad (D.3)$$

$$\text{where } S_x = (N-n) \sum x_i^2 - (\sum x_i)^2$$

$$\text{and } S_{xn} = (N-n) \sum x_{i+n}^2 - (\sum x_{i+n})^2$$



Positive part of cross covariance:

$$C(n) = \frac{1}{N-n} \sum x_i y_{i+n} - \frac{1}{(N-n)^2} \sum x_i \sum y_{i+n} \quad (D.4)$$

Negative part of cross covariance:

$$D(n) = \frac{1}{N-n} \sum y_i x_{i+n} - \frac{1}{(N-n)^2} \sum y_i \sum x_{i+n} \quad (D.5)$$

In-phase cross covariance:

$$E(n) = \frac{D(n) + C(n)}{2} \quad (D.6)$$

Out-of-phase cross covariance:

$$F(n) = \frac{D(n) - C(n)}{2} \quad (D.7)$$

In-phase cross correlation:

$$S(n) = \frac{(N-n)^2 E(n)}{\sqrt[4]{S_x S_{xn} S_y} S_{yn}} \quad (D.8)$$

Out-of-phase cross correlation:

$$T(n) = \frac{(N-n)^2 F(n)}{\sqrt[4]{S_x S_{xn} S_y} S_{yn}} \quad (D.9)$$

where  $S_x$ ,  $S_{xn}$ ,  $S_{yn}$  are defined as in Equation (D.3)

Primary spectrum estimate:

$$X(n) = \frac{d}{M} [ \sum e(k) A(k) \cos \frac{\pi n k}{M} + A(0) ] \quad (D.10)$$



Secondary spectrum estimate:

$$Y(n) = \frac{d}{M} [\sum e(k) B(k) \cos \frac{\pi nk}{M} + B(0)] \quad (D.11)$$

Co-spectrum estimate:

$$Z(n) = \frac{d}{M} [\sum e(k) E(k) \cos \frac{\pi nk}{M} + E(0)] \quad (D.12)$$

Quaspectrum estimate:

$$W(n) = \frac{d}{M} [\sum e(k) F(k) \sin \frac{\pi nk}{M}] \quad (D.13)$$

where  $F(0)$  always = 0

For the previous four formula:

$$e(k) = 1 + \cos \frac{\pi k}{M} \text{ for Hanning smoothing}$$

and  $d = \frac{1}{2}$  for  $n = 0$  and  $n = M$

$d = 1$  for all other  $n$ .

Furthermore,

Cross spectral amplitude:

$$G(n) = \sqrt{Z^2(n) + W^2(n)} \quad (D.14)$$

Coherence coefficient:

$$R(n) = \frac{G(n)}{\sqrt{X(n) Y(n)}} \quad (D.15)$$

Phase lead (secondary over primary):

$$\text{PL}(n) = \tan^{-1} \frac{W(n)}{Z(n)} \quad (D.16)$$



Frequency:

$$\text{FREQ}(n) = \frac{n}{2Mt} \quad (\text{D.17})$$

### Statistical Formulae

The summations of Equation (D.18) to (D.20) are  $i = 1, 2, 3 \dots N$ .  $N$  is the total number of points in the time series.

Mean:

$$M = \frac{1}{N} \sum x_i \quad (\text{D.18})$$

Standard deviation:

$$SD = \left( \frac{1}{N} \sum (x_i - M)^2 \right)^{\frac{1}{2}} \quad (\text{D.19})$$

Skewness:

$$SK = \frac{1}{N SD^3} \sum (x_i - M)^3 \quad (\text{D.20})$$



## APPENDIX E

## SPECTRA

The confidence limit factors for the following spectra are given below:

$$\text{Upper confidence limit, } U = C_u \bar{S}$$

$$\text{Lower confidence limit, } L = C_1 \bar{S}$$

The factors  $C_u$  and  $C_1$  are given in the Table XIII.  $\bar{S}$  is the running mean of the spectra, S.

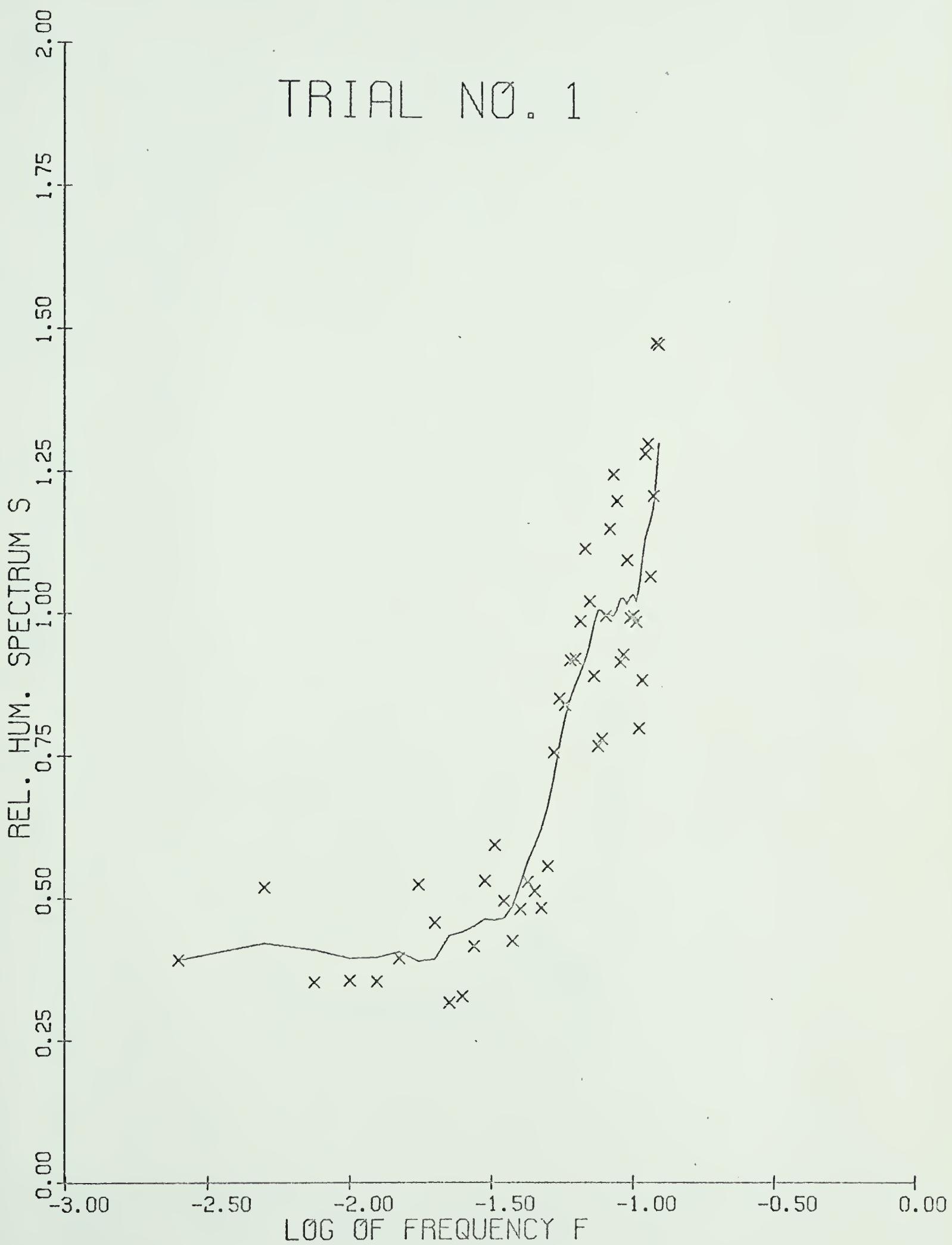
Table XIII - Multiplication Constants for Upper and Lower Confidence Limits.

No.	Type	degrees of freedom	$C_u$	$C_1$
1	RH	19	7.76	0.129
8	RH	8	3.78	0.265
11	RH	14	5.81	0.172
3	RH	7	3.47	0.288
6	RH	9	4.10	0.244
9	RH	9	4.10	0.244
7	RH	16	6.56	0.153
10	RH	36	16.88	0.060
13	RH	17	8.36	0.120
7L	RH	7	3.47	0.288
9	q	9	4.10	0.244
10	T	29	12.47	0.080

These factors apply to the smooth spectra,  $\bar{S}$ , plotted in the following graphs. For an individual peak (trough) to be significant at the 10% level its value must be larger than U (less than L). The spectra plotted are  $S = 2.303 nS^*/s^2$  and the frequency is equal to  $\log_{10} nz/V$  where n is in cycles per sec.

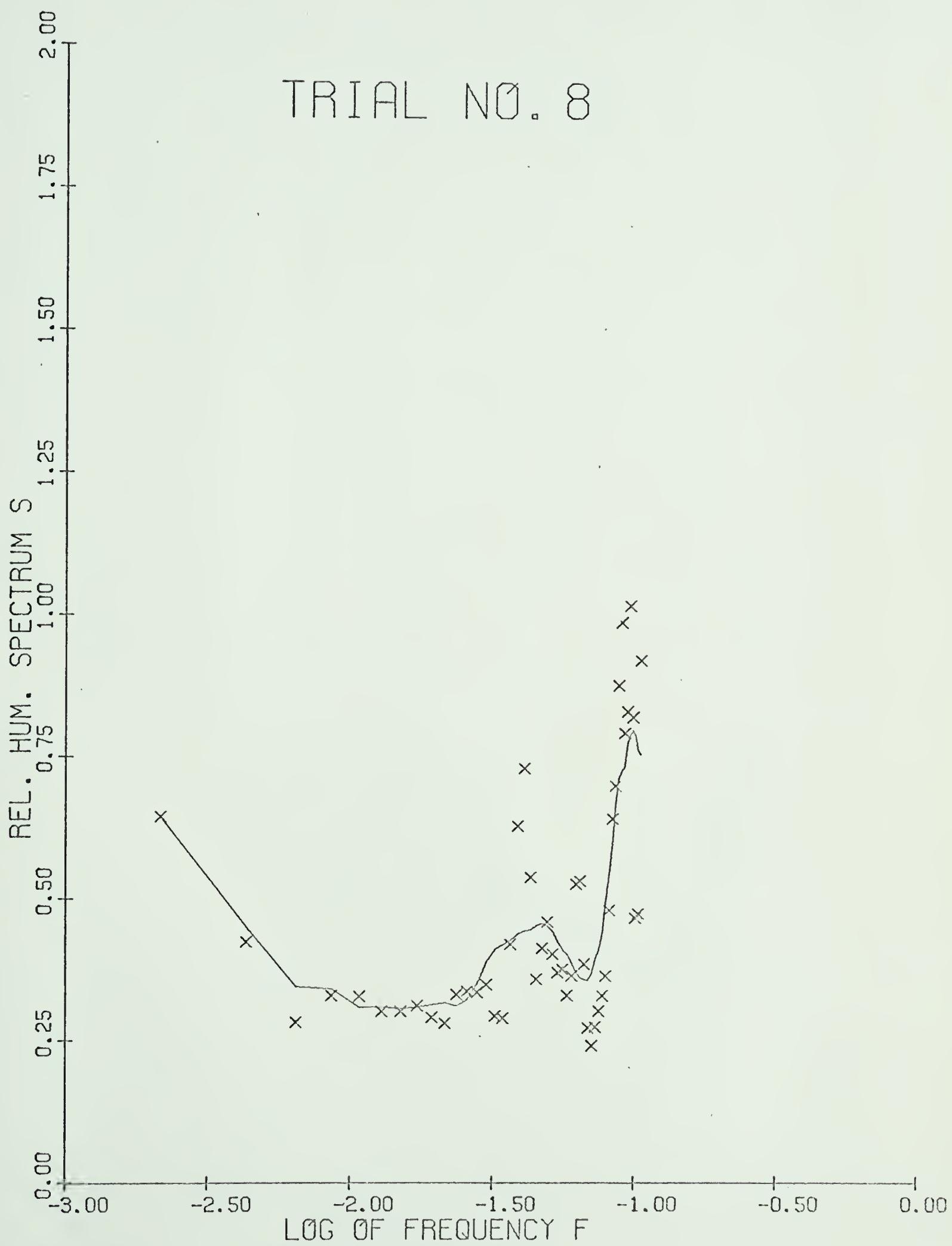


## TRIAL NO. 1

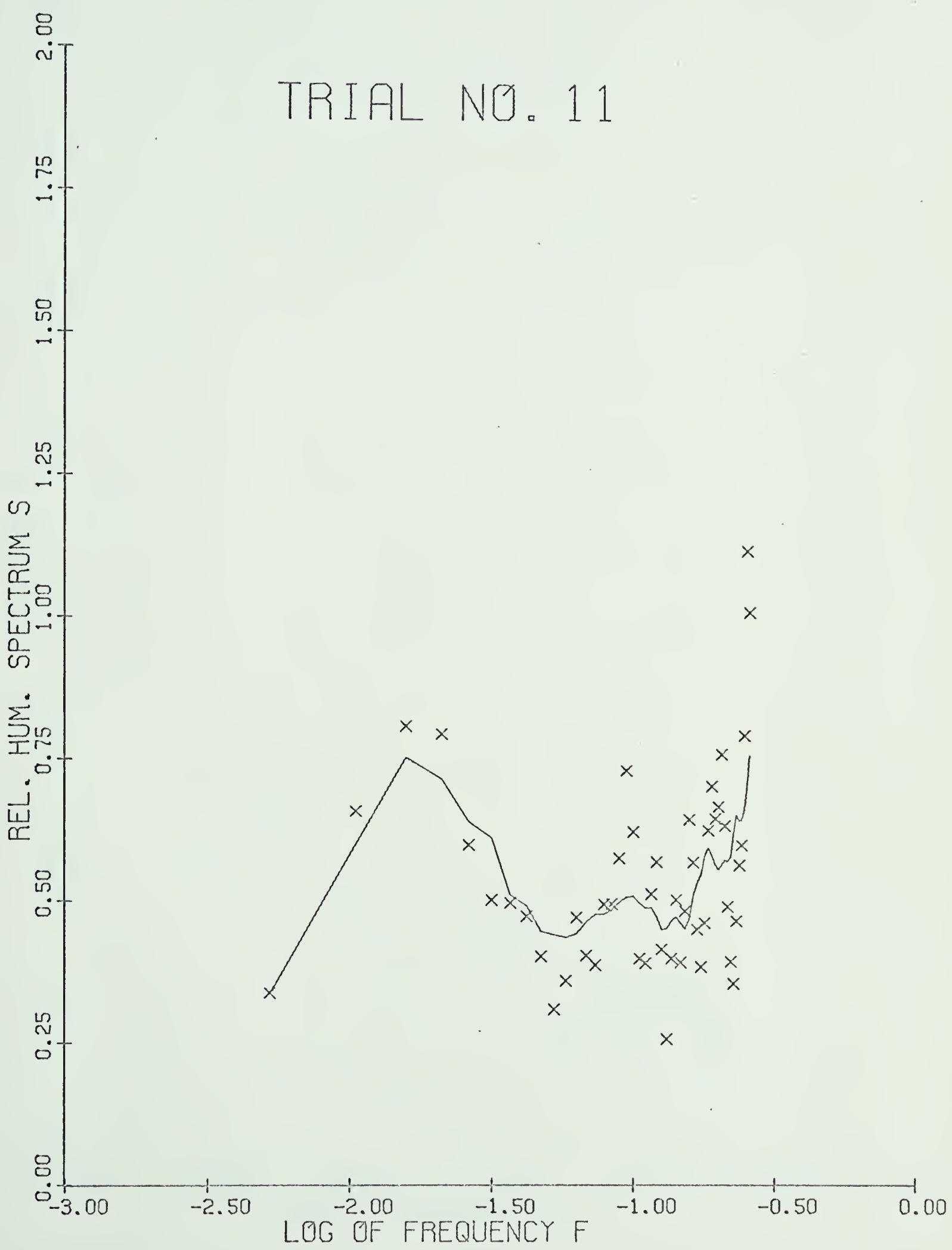




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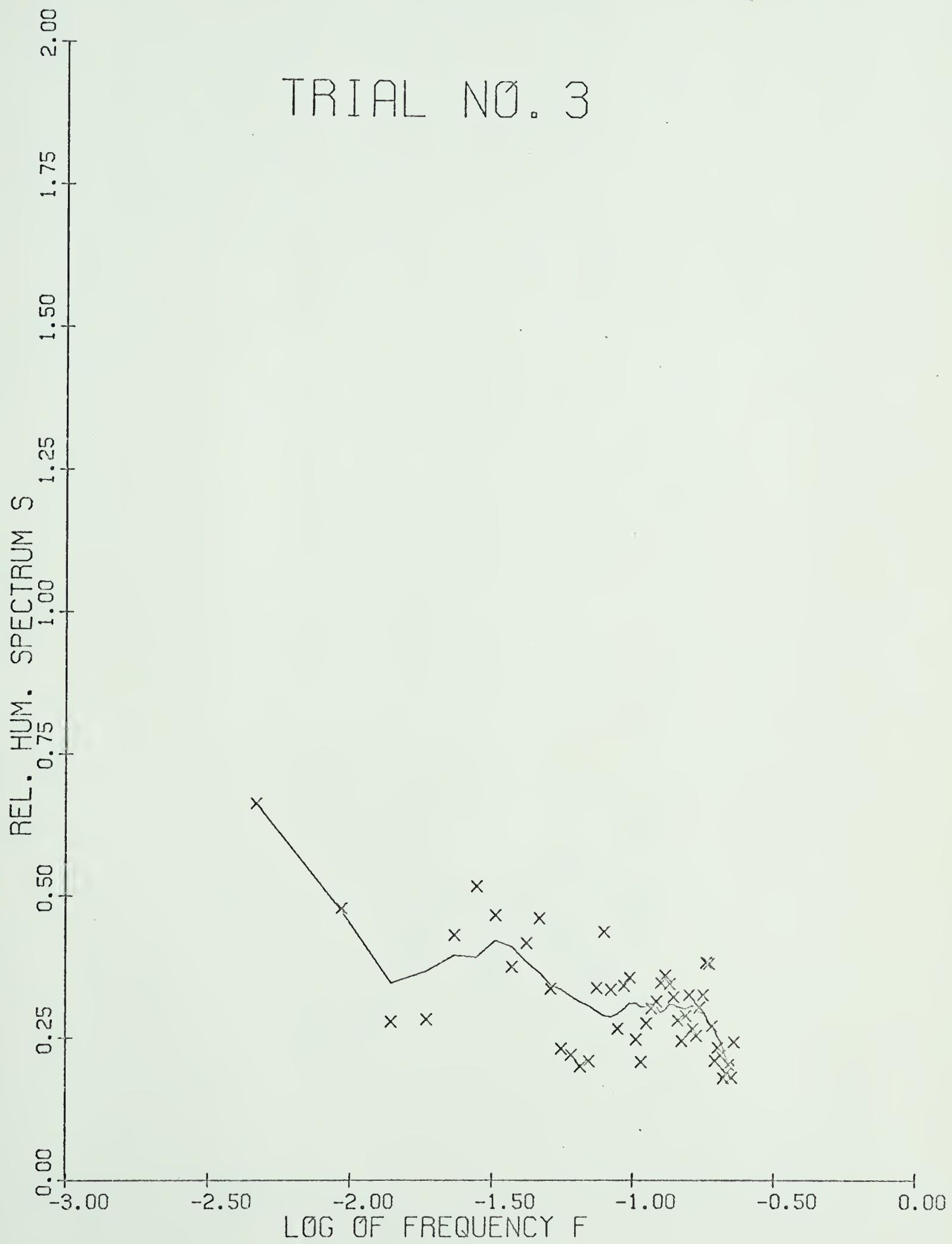






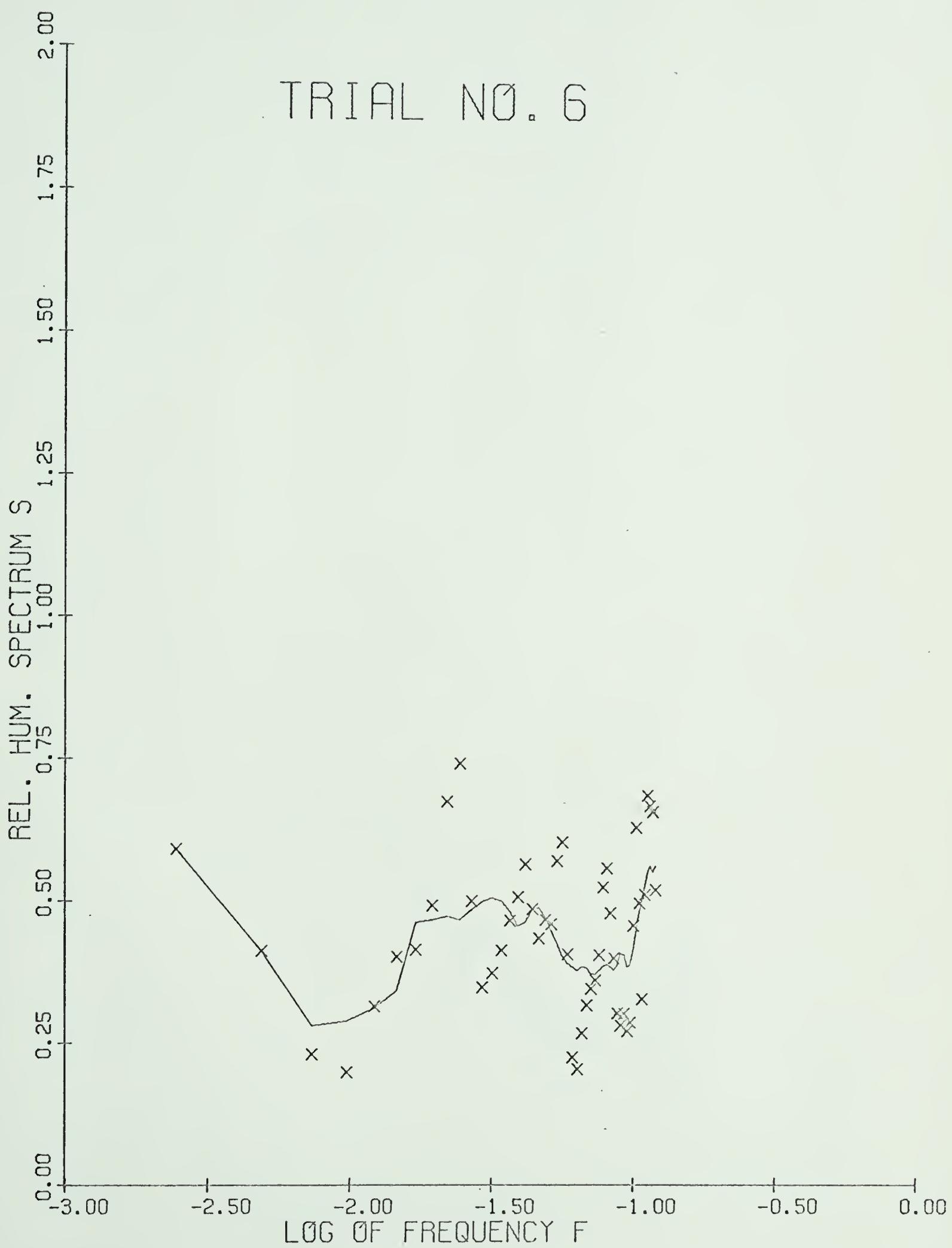


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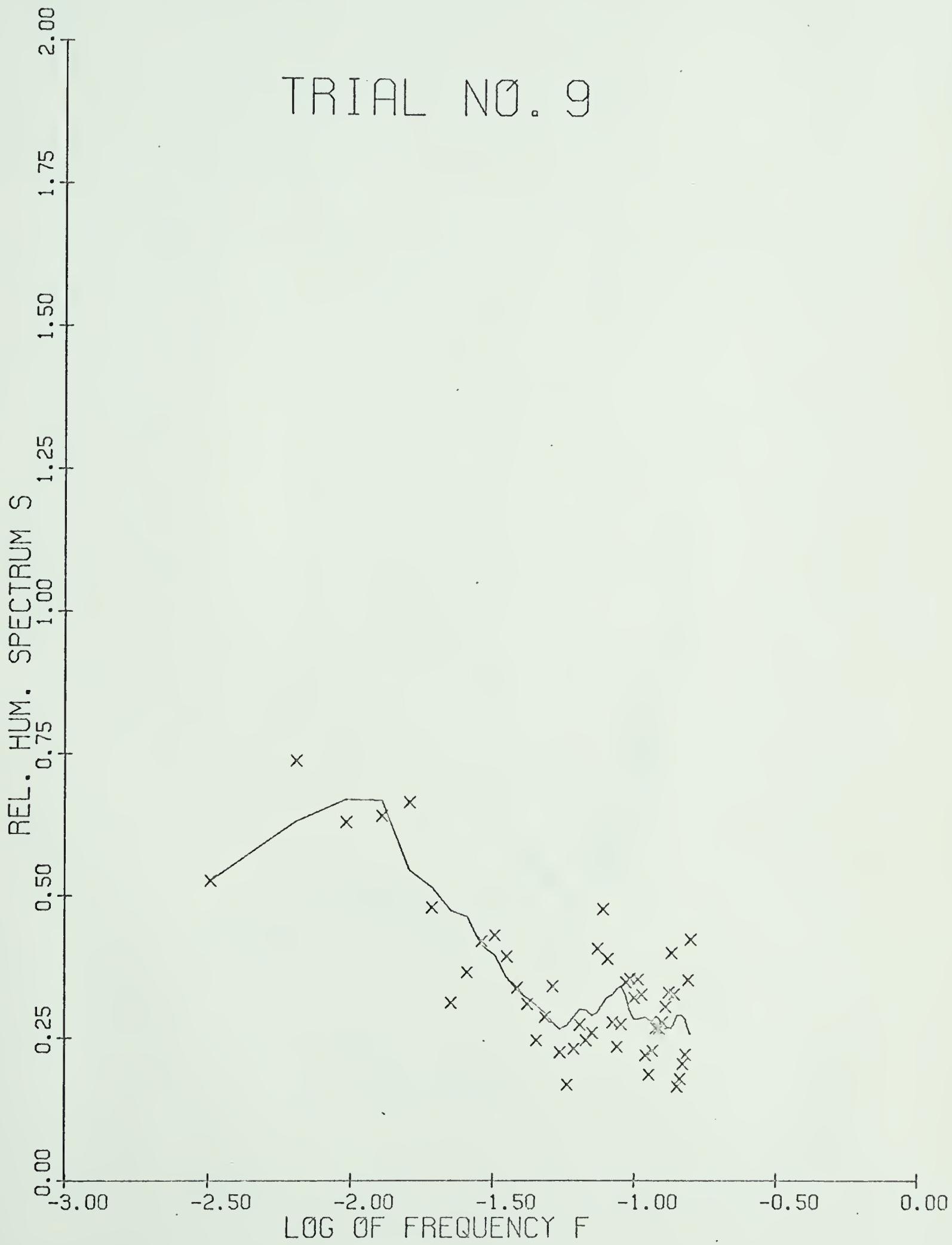


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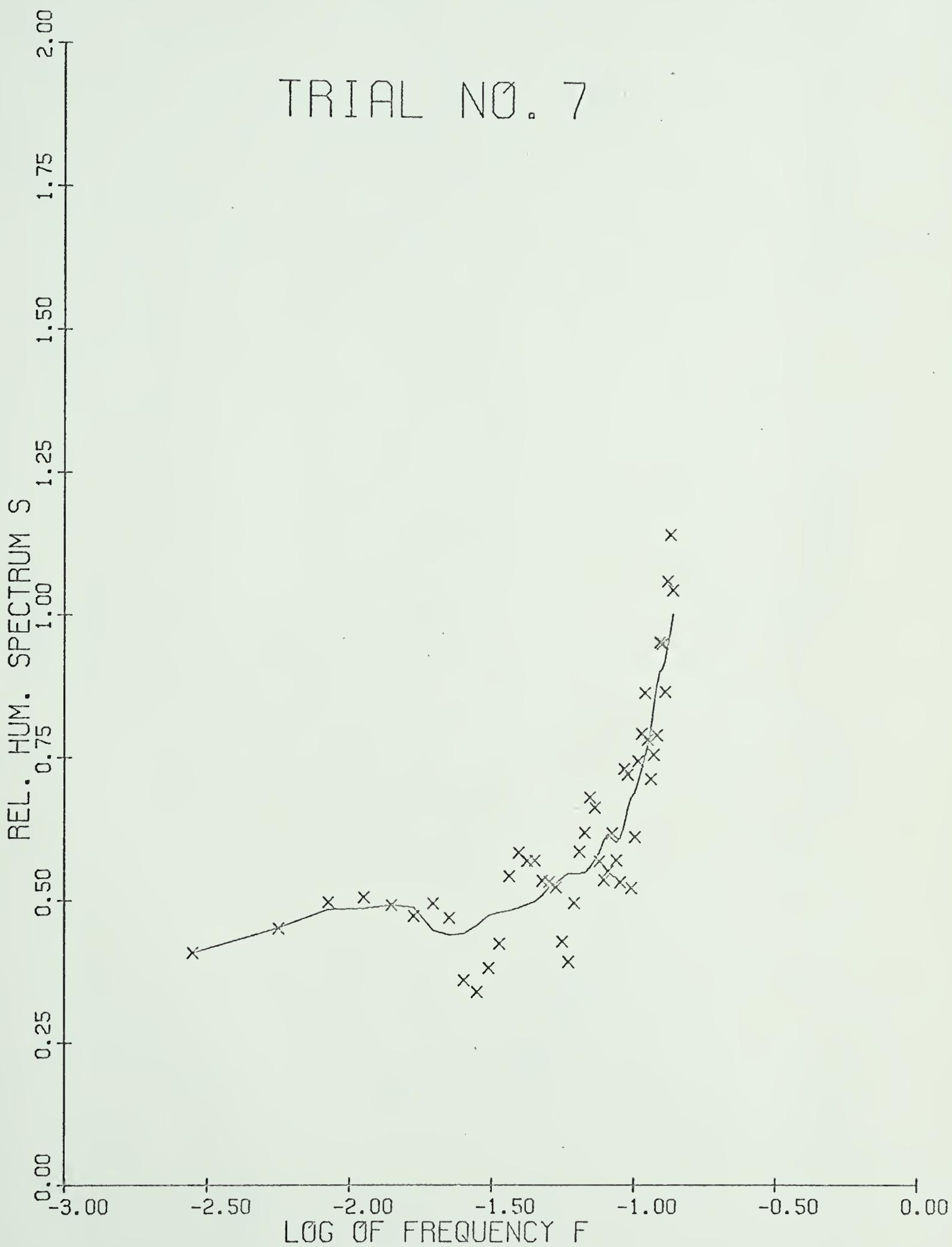


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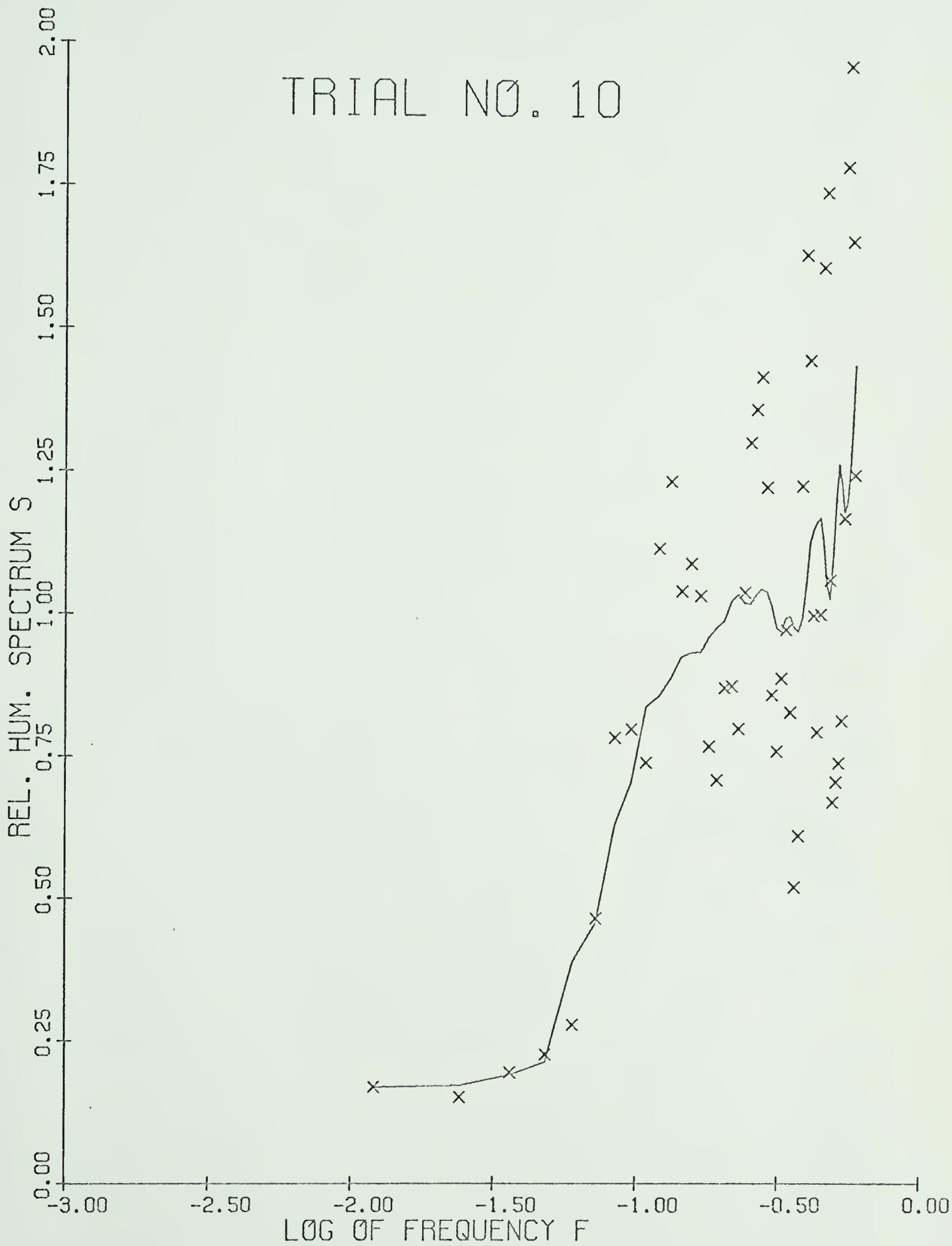




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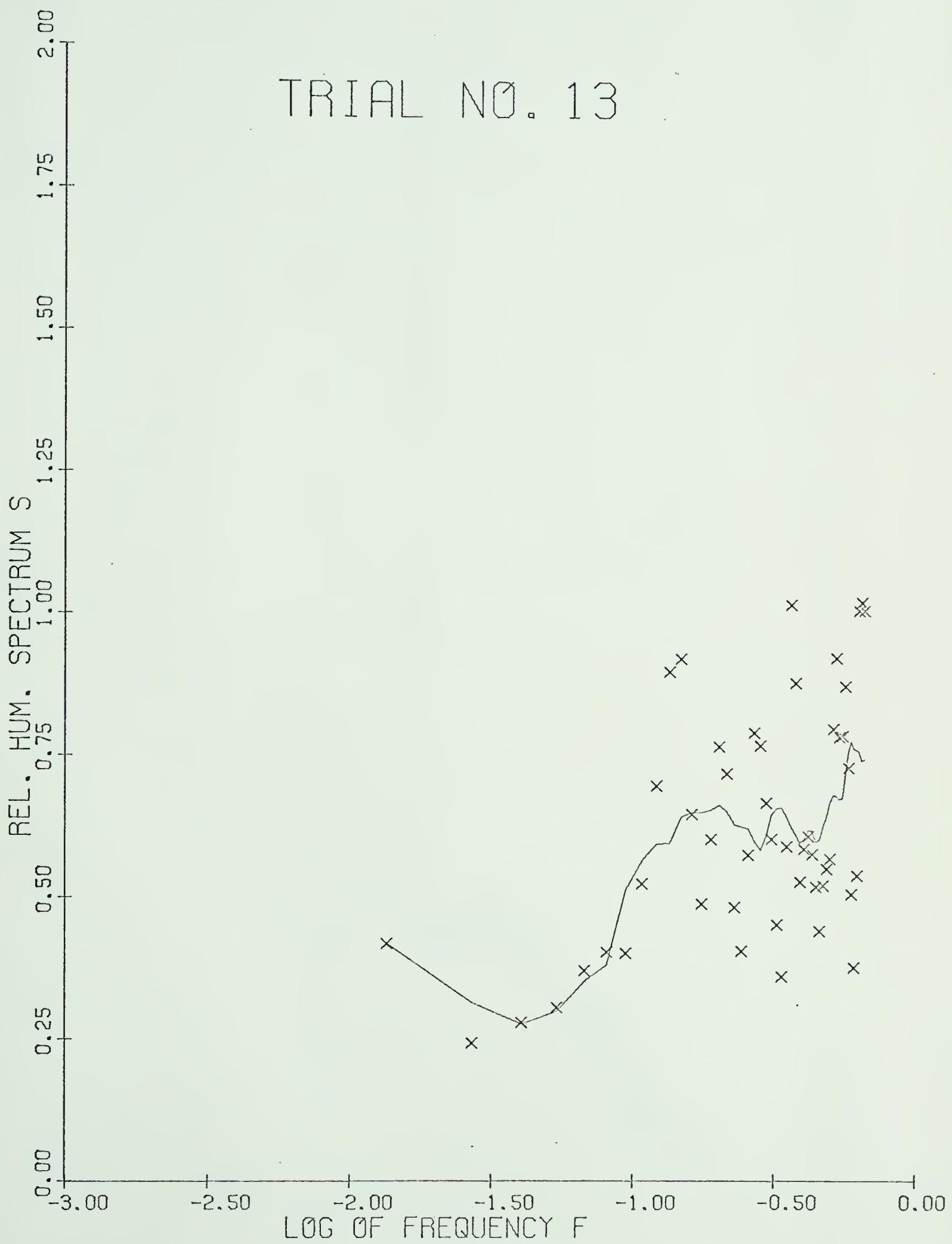






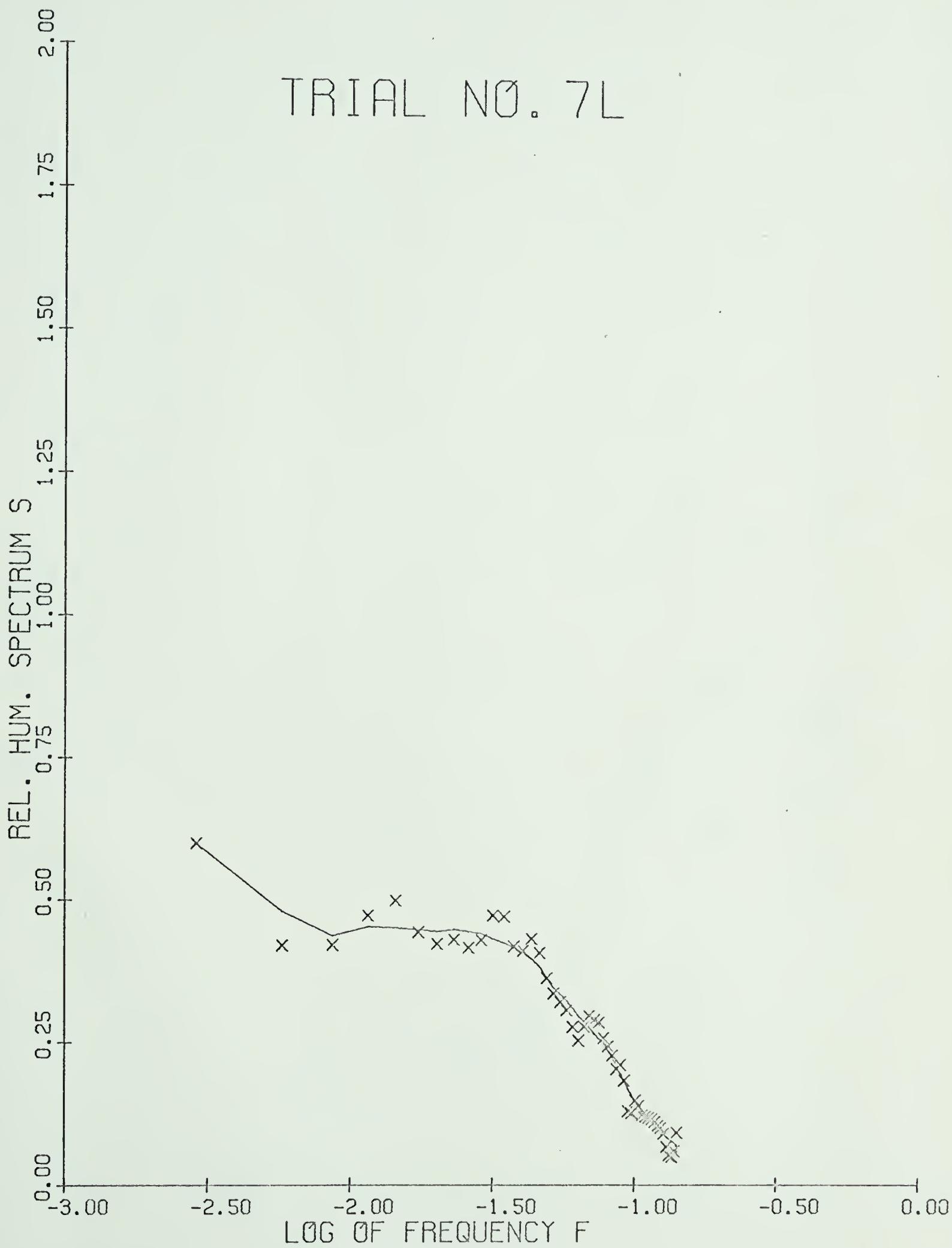


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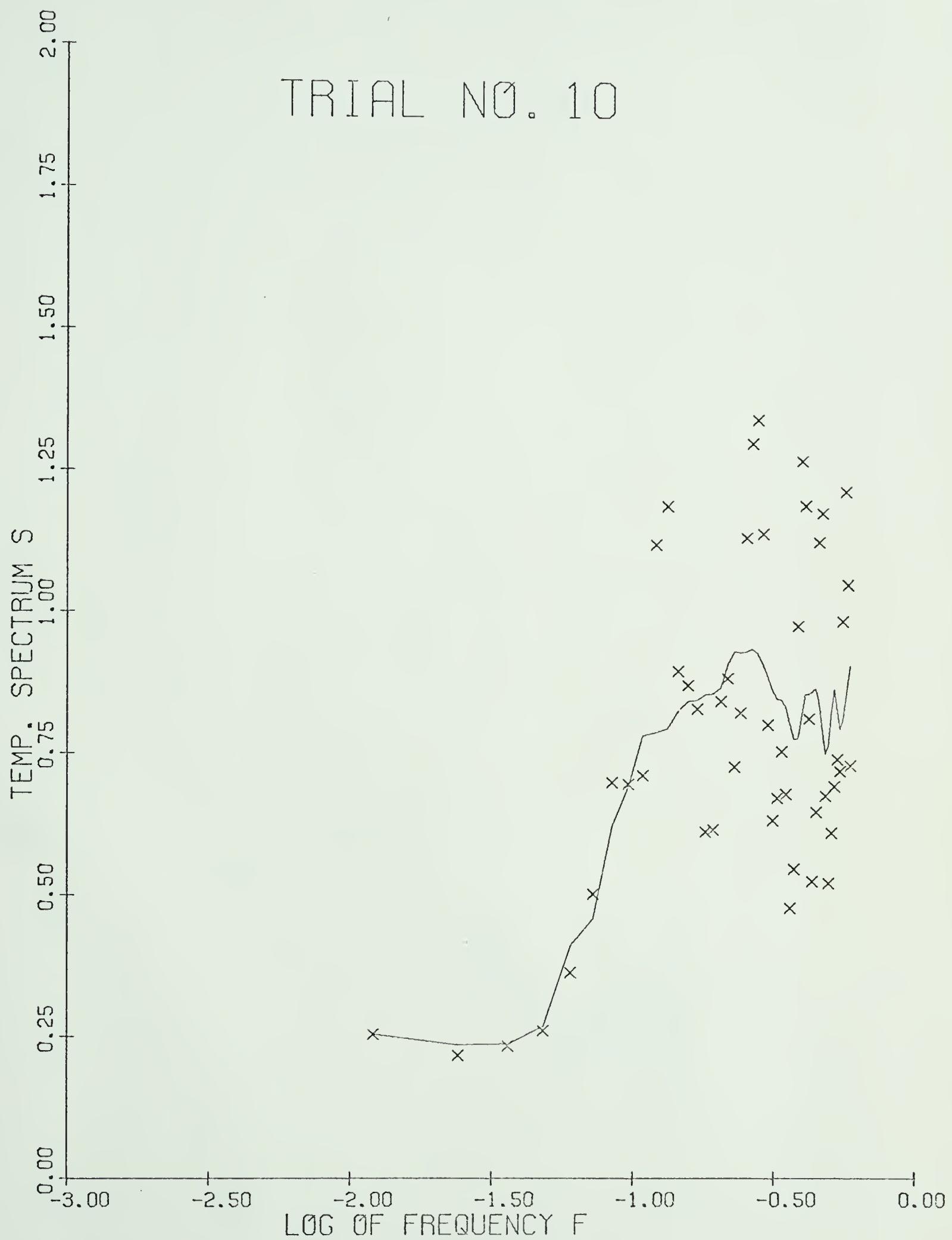


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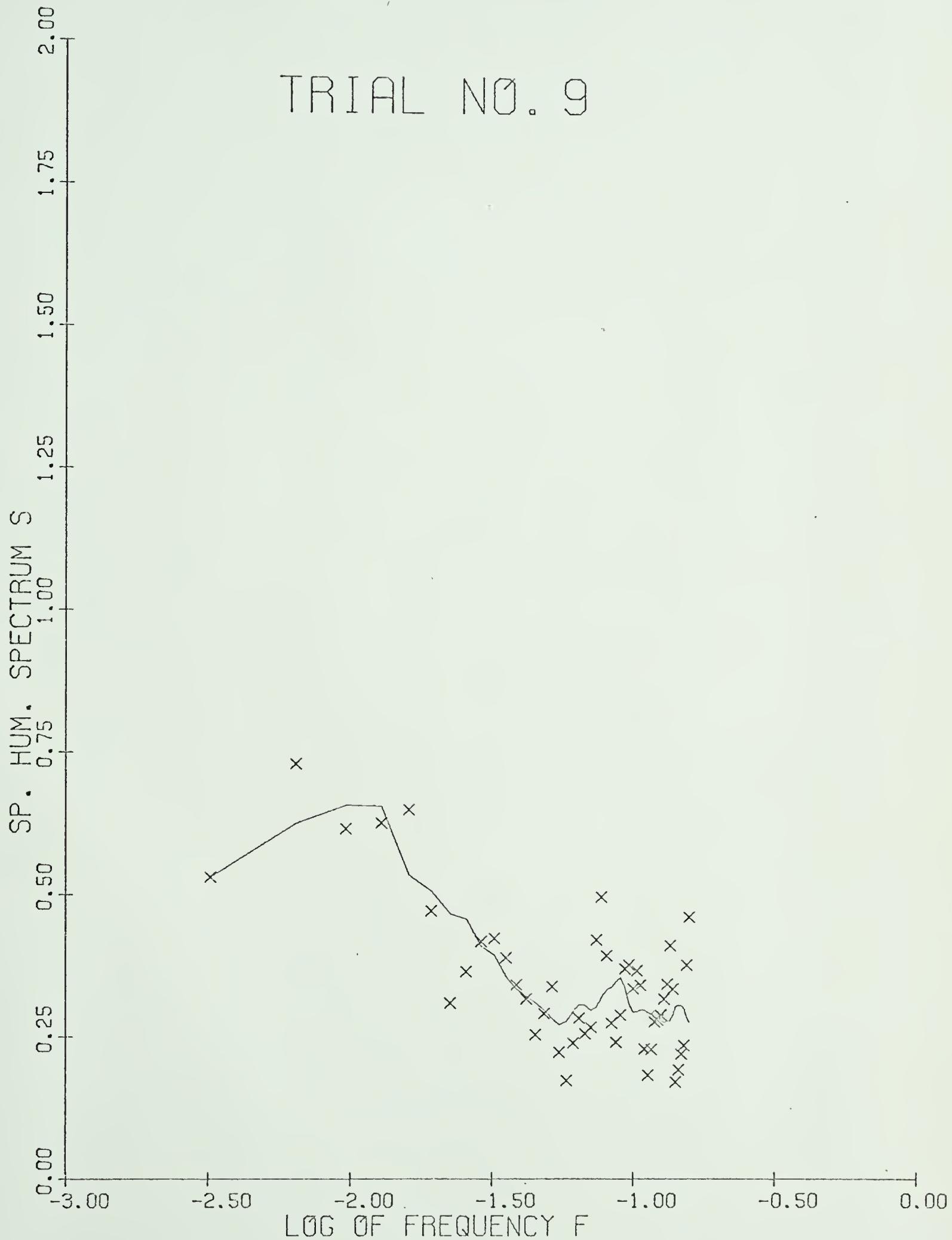


## TRIAL NO. 10





## TRIAL NO. 9











**B29896**